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NDA MATHS MOCK TEST - 196 (SOLUTION)

1. (C) The given circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$

The cetres of the given circles are $c_1(5, 0)$ and $c_2(0, 0)$ and their radii are $r_1 = 3$ anf $r_2 = r$. For the circles to intersect at two distinct points.

$|r_1 - r_2| < c_1c_2 < (r_1 + r_2)$

$\Rightarrow |3 - r| < 5 < (r + 3)$

$\Rightarrow r - 3 < 5 < r + 3$

$\Rightarrow r < 8$ and $r > 2 \Rightarrow 2 < r < 8$

2. (B) The circle $2(x^2 + y^2) + x - y + 5 = 0$

$\Rightarrow x^2 + y^2 + \frac{x}{2} - \frac{y}{2} + \frac{5}{2} = 0$

Length of tangent from (0, 0) to this circle

$= \sqrt{0+0+0+0+\frac{5}{2}} = \sqrt{\frac{5}{2}}$

3. (C) The centres and radii of the circles are as centres $C_1\left(\frac{1}{2},0\right)$, $C_2\left(\frac{-1}{2},0\right)$

radii $r_1 = \frac{1}{2}$, $r_2 = \frac{1}{2}$

It is clear that $c_1c_2 = r_1 + r_2$

So the circle touch each other externally. Hence, there will be 3 common tangents.

4. (C) The ellipse equation $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ ...(i)

Let P (acosθ, b sinθ) be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ ...(i)

Then equation of normal at point P is

$ax \sec\theta - by \operatorname{cosec}\theta = (a^2 - b^2)$

...(ii)

It meets the x-axis at $R\left(\frac{a^2-b^2}{a}\cos\theta,0\right)$

and y-axis at $S\left(0,\frac{a^2-b^2}{a}\sin\theta\right)$

Now, $(PR)^2=\left\{a\cos\theta-\frac{a^2-b^2}{a}\cos\theta\right\}^2 + b^2\sin\theta$

$= \left\{\frac{b^2}{a}\cos\theta\right\}^2 + b^2\sin^2\theta$

$= \frac{b^2}{a^2}(b^2\cos^2\theta + a^2 \sin^2\theta)$

and $(PS)^2 = \frac{a^2}{b^2}(b^2\cos^2\theta + a^2\sin^2\theta)$

$$\frac{(PR)^2}{(PS)^2} = \frac{\frac{b^2}{a^2}(b^2\cos^2\theta+a^2\sin^2\theta)}{\frac{a^2}{b^2}(b^2\cos^2\theta+a^2\sin^2\theta)} = \frac{b^4}{a^4}$$

$\Rightarrow \frac{PR}{PS} = \frac{b^2}{a^2}$

$\Rightarrow (PR) : (PS) = b^2 : a^2$

5. (C) The ellipse equation $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$

and line equation $\frac{x}{a}+\frac{y}{b} = \sqrt{2}$

Let point P($a$cosθ, $b$sinθ) at the ellipse (i) then equation of tangent at point P is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

On comparing equation (ii) and (iii) we have $\cos\theta = \sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

6. (A) Let $y = m_1x$ and $y = m_2x$ be a pair of conjugate diameters of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

and let P(a cosθ, b sinθ) and Q(a cosϕ, b sinϕ) be ends of these two diameters.

Then $m_1m_2 = -\frac{b^2}{a^2}$

$\Rightarrow \frac{(b\sin\theta-0)}{(a\cos\theta-0)}\times\frac{(b\sin\phi-0)}{(a\cos\phi-0)} = -\frac{b^2}{a^2}$

$\Rightarrow \sin\theta\sin\phi = -\cos\theta.\cos\phi \Rightarrow \cos\theta.\cos\phi + \sin\theta.\sin\phi = 0$

$\Rightarrow \cos(\theta + \phi) = 0$

$\Rightarrow \theta - \phi = \pm\frac{\pi}{2}$

7. (B) The ellipse equation $2x^2 + 5y^2 = 20$

$\Rightarrow \frac{x^2}{10} + \frac{y^2}{4} = 1$

The equation of chord bisected at the point (2, 1) is $T = S_1$

$\Rightarrow \frac{x}{10} \times 2 + \frac{y}{4} \times 1 - 1 = \frac{(2)^2}{10} + \frac{(1)^2}{4} = 1$

$\Rightarrow \frac{x}{5} + \frac{y}{4} = \frac{2}{5} + \frac{1}{4} = \frac{13}{20} \Rightarrow 4x + 15y = 13$

8. (A) The ellipse equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Compare this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have

$a^2 = 16$ $b^2 = 9$

Now, eccentricity of the ellipse

$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

The coordinates of the foci are $(\pm ae, 0)$ or $(\pm\sqrt{7}, 0)$

So, radius of the circle = distance between $(\pm\sqrt{7}, 0)$ and (0, 3)

$= \sqrt{7+9} = 4$

9. (B) The combined equation of the asymptotes is $(3x - 4y + 7)(4x + 3y + 1) = 0$

So the equation of hyperbola

$(3x - 4y + 7)(4x + 3y + 1) + 1 = 0$ ...(i)

Since it passes through origin, then

$\therefore 7 + 1 = 0 \Rightarrow \lambda = -7$

Put this value of λ in equation (i), we get the required equation of Hyperbola

$12x^2 - 7xy - 12y^2 + 31x + 17y = 0$

10. (B) Ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

Foci of ellipse $\left(\pm\sqrt{16 - b^2}, 0\right)$

Hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

Foci of hyperbola $\left(\pm\sqrt{\frac{144}{25} + \frac{81}{25}}, 0\right)$

i.e., $(\pm 3, 0)$

According to question

$\sqrt{16 - b^2} = 3 \Rightarrow 16 - b^2 = 9 \Rightarrow b^2 = 7$

11. (A) Let equation of tangent to the parabola $y^2 = 8x$ is $y = mx + \frac{2}{m}$ ...(i)

Equation of hyperbola ...(ii)

$3x^2 - y^2 = 3$

Eliminating y between (i) and (ii), we have

$3x^2 - \left(mx + \frac{2}{m}\right)^2 = 3$

$\Rightarrow (3 - m^2)x^2 - 4x - \left(\frac{4}{m^2} + 3\right) = 0$ ...(iii)

Since equation (i) touch equation (ii) (Hyperbola) so roots of eq. (3) will be real and equal so-

$B^2 - AC = 0$

$\Rightarrow (-4)^2 - 4(3 - m^2)\left[-\left(\frac{4}{m^2} + 3\right)\right] = 0$

$\Rightarrow m^4 - 3m^2 - 4 = 0$

$\Rightarrow m^4 - 4m^2 + m^2 - 4 = 0$

$\Rightarrow (m^2 - 4)(m^2 + 1) = 0$

$\Rightarrow m^2 - 4 = 0$ or $m^2 + 1 = 0$

$\Rightarrow m^2 = 4, m^2 = -1$

$\Rightarrow m = \pm 2$

$\therefore$ equation of common tangent

$y = \pm(2x + 1)$

12. (A) Ellipse $3x^2 + 4y^2 = 12$

$\Rightarrow \frac{3x^2}{12} + \frac{4y^2}{12} = 1$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$ ...(i)

hyperbola, transverse axis = $2\sin\theta$

$a = \sin\theta$

Foci of ellipse $(\pm 1, 0)$

Now, $\sqrt{a^2 + b^2} = \pm 1$

$\Rightarrow a^2 + b^2 = 1$

$\Rightarrow \sin^2\theta + b^2 = 1 \Rightarrow b^2 = \cos^2\theta$

$\therefore$ Equation of hyperbola

$\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

$\Rightarrow x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$

13. (A) $I = \int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$

put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$

$I = \int 2\cos t.dt \Rightarrow I = 2\sin t + C$

$I = 2\sin\sqrt{x} + C$

14. (A) Let I = $\int \frac{(\cos 4x+1)dx}{(\cot x-\tan x)}$

$\Rightarrow$ I = $\int \frac{2\cos^2 2x}{\left(\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}\right)}dx$

$\Rightarrow$ I = $\int \frac{2\cos^2 2x}{\frac{(\cos^2 x-\sin^2 x)}{\sin x.\cos x}}dx$

$\Rightarrow$ I = $\int \frac{2\cos^2 2x}{\frac{\cos 2x}{\sin x.\cos x}}dx$

$\Rightarrow$ I = $\int 2\cos 2x.(\sin x.\cos x)dx$

$\Rightarrow$ I = $\int \cos 2x(2\sin x.\cos x)dx$

$\Rightarrow$ I = $\int \cos 2x.\sin 2x\,dx$

$\Rightarrow$ I = $\frac{1}{2}\int \sin 4x\,dx$

$\Rightarrow$ I = $-\frac{1}{8}\cos 4x$ + C ...(i)

Now, it is given that

I = P $\cos 4x$ + C ...(ii)

On comparing equation (i) and (ii), we get

P = $-\frac{1}{8}$

15. (A) $\int \cos\sqrt{x}\,dx$

put $x = t^2 \Rightarrow dx = 2t.dt$

$\Rightarrow \int 2t.\cos t.dt \Rightarrow 2\int t.\cos t\,dt$

$\Rightarrow 2\left[t.\sin t-\int \sin t\,dt\right]$

$\Rightarrow 2[t.\sin t + \cos t]$ + C

$\Rightarrow 2\left[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}\right]$ + C

16. (A) Let I = $\int x^3.e^{x^2}dx = \int x.x^2.e^{x^2}dx$

put $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{1}{2}dt$

$\Rightarrow$ I = $\int \frac{1}{2}t.e^t dt$

$\Rightarrow$ I = $\frac{1}{2}[t.e^t - e^t]$ + C

$\Rightarrow$ I = $\frac{1}{2}[x^2.e^{x^2} - e^{x^2}]$ + C

$\Rightarrow$ I = $\frac{1}{2}e^{x^2}(x^2-1)$ + C

17. (A) I = $\int \frac{e^{2x}+1}{(e^{2x}-1)}dx = \int \frac{(e^x+e^{-x})}{(e^x-e^{-x})}dx$

Put $e^x - e^{-x} = t \Rightarrow (e^x + e^{-x}) = dt$

$\Rightarrow$ I = $\int \frac{dt}{t} \Rightarrow$ I = $\log t$ + C

$\Rightarrow$ I = $\log|e^x - e^{-x}|$ + C

18. (A) Let I = $\int \sec^2\frac{x}{2}.\operatorname{cosec}^2\frac{x}{2}\,dx$

I = $\int \sec^2\frac{x}{2}\left(1+\cot^2\frac{x}{2}\right)dx$

I = $\int \left(\sec^2\frac{x}{2}+\sec^2\frac{x}{2}.\cot^2\frac{x}{2}\right)dx$

I = $\int \sec^2\frac{x}{2}dx + \int \sec^2\frac{x}{2}.\cot^2\frac{x}{2}dx$

I = 2 $\tan\frac{x}{2} + \int \frac{1}{\cos^2\frac{x}{2}}.\frac{\cos^2\frac{x}{2}}{\sin^2\frac{x}{2}}dx$

I = 2 $\tan\frac{x}{2} + \int \operatorname{cosec}^2\frac{x}{2}dx$

I = 2$\tan\frac{x}{2}$ – 2$\cot\frac{x}{2}$ + C

I = $2\left(\tan\frac{x}{2}-\cot\frac{x}{2}\right)$ + C

19. (A) Given that $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ $(z \neq -1)$

Now we know that $z\bar{z} = |z|^2$

$\Rightarrow z\bar{z} = 1$ (for $|z| = 1$)

$\therefore \omega = \left(\frac{z-1}{z+1}\right)\times\left(\frac{\bar{z}+1}{\bar{z}+1}\right) = \frac{z\bar{z}+z-\bar{z}-1}{z\bar{z}+z+\bar{z}+1} = \frac{2iy}{2+2y}$

[$\because z\bar{z}$ = 1 and taking $z = x + iy$ so that $z + \bar{z}$ + = $2x$ and $z - \bar{z} = 2iy$]

$\Rightarrow$ Re(ω) = 0

20. (B) $(1 + \omega^2)^n = (1 + \omega^4)^n$

$\Rightarrow (-\omega)^n = (1+\omega)^n = (-\omega^2)^n \Rightarrow \omega^n = 1 \Rightarrow n = 3$

21. (B) $\begin{vmatrix}1 & 1 & 1\\ 1 & -1-\omega^2 & \omega^2\\ 1 & \omega^2 & \omega^4\end{vmatrix} \Rightarrow \begin{vmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}$

$C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix}1 & 0 & 0\\ 1 & \omega-1 & \omega^2-1\\ 1 & \omega^2-1 & \omega-1\end{vmatrix}$

$\Rightarrow 1[(\omega-1)^2 - (\omega^2-1)^2]$

$\Rightarrow 3\omega^2 - 3\omega \Rightarrow 3\omega(\omega-1)$

22. (D) $\because \frac{w-wz}{1-z}$ is purely real

$\therefore \overline{\left(\frac{w-\overline{w}z}{1-z}\right)} = \left(\frac{w-\overline{w}z}{1-z}\right)$

$\Rightarrow \frac{\overline{w}-\overline{w}z}{1-\overline{z}} = \frac{\overline{w}-\overline{w}z}{1-z}$

$\Rightarrow \overline{w}-\overline{w}z-w\overline{z}+wz\overline{z} = w-w\overline{z}+\overline{w}z\overline{z}$

$\Rightarrow w-\overline{w} = (w-\overline{w})|z|^2$

$\Rightarrow |z|^2 = 1 (\because w = \alpha + i\beta$ and $\beta \neq 0)$

$\Rightarrow |z| = 1$ also given $z \neq 1$

$\therefore$ The required set is $\{z : |z| = 1, z \neq 1\}$

23. (B) We know that

$\Rightarrow -\sqrt{a^2+b^2} \le \alpha\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$

$\Rightarrow -\sqrt{74} \le 7\cos x + 5\sin x \sqrt{74}$

$\Rightarrow -\sqrt{74} \le 2k+1 \le \sqrt{74} \Rightarrow -8.6 \le 2k+1 \le 8.6$

$\Rightarrow -4.8 \le k \le 3.8$

(considering only itegral values)

Hence k can take 8 integral values.

24. (B) Given that $\sin\theta = \frac{1}{2}$ and $\cos\phi = \frac{1}{3}$ and θ and ϕ both are acute angles

$\therefore \theta = \frac{\pi}{6}$ and $0 < \frac{1}{3} < \frac{1}{2}$

or $\cos\frac{\pi}{2} < \cos\phi < \cos\frac{\pi}{3}$ or $\frac{\pi}{3} < \phi < \frac{\pi}{2}$

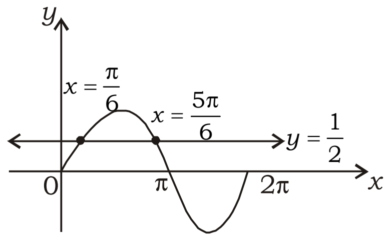
$\therefore \frac{\pi}{3}+\frac{\pi}{6} < \theta + \phi < \frac{\pi}{2}+\frac{\pi}{6}$ or $\frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$

$\Rightarrow \pi + \phi \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

25. (A) $2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow (\sin\theta - 2)(2\sin\theta - 1) > 0$

$\Rightarrow \sin\theta < \frac{1}{2}$ $[-1 \le \sin\theta \le 1]$



From graph, we get

$x \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

26. (B) $\because \theta \in \left(0, \frac{\pi}{4}\right) \Rightarrow \tan\theta < 1$ and $\cot\theta > 1$

Let $\tan\theta = 1 - x$ and $\cot\theta = 1 + y$

Where $x, y > 0$ and are very small, then

$\therefore t_1 = (1-x)^{1-x}, t_2 = (1-x)^{1+y}, t_3 = (1+y)^{1-x}, t_4 = (1+y)^{1+y}$

Clearly, $t_4 > t_3$ and t_1 also, $t_3 > t_1$

Thus $t_4 > t_3 > t_1 > t_2$

27. (A) Curve $y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x}$

at point (0, 1) $\frac{dy}{dx} = 2e^0 = 2$

The equation of tangent at point (0, 1)

$y - 1 = 2(x - 0) \Rightarrow y - 2x = 1$

At x-axis, $y = 0 \Rightarrow x = -\frac{1}{2}$

So required point $\left(-\frac{1}{2}, 0\right)$

28. (A) Curve $x^3 + y^3 = 6xy$

On differentiating w.r.t. 'x'

$\Rightarrow 3x^2 + 3y^2\frac{dy}{dx} = 6\left(y + x.\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{2y - x^2}{(y^2 - 2x)}$

At point (3, 3), $\frac{dy}{dx} = \frac{dy}{dx}\ \frac{2\times 3 - (3)^2}{(3)^2 - 2(3)} = -1$

Slope of normal $= -\frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{1}{(-1)} = 1$

Equation of normal at point (3, 3)

$y - 3 = 1.(x - 3) \Rightarrow y - x = 0$

29. (C) Differential equation

$xdy - ydx = (x^3 + xy^2)dx$

$\Rightarrow xdy - ydx = x(x^2 + y^2)dx$

$\Rightarrow \dfrac{xdy - ydx}{x^2 + y^2} = x\,dx$

$\Rightarrow \dfrac{d}{dx}\left(\tan^{-1}\dfrac{x}{y}\right) = x\,dx$

$\Rightarrow \int \dfrac{d}{dx}\left(\tan^{-1}\dfrac{x}{y}\right) = \int x\,dx$

$\Rightarrow \tan^{-1}\dfrac{x}{y} = \dfrac{x^2}{2} + \dfrac{c}{2}$

$\Rightarrow 2\tan^{-1}\dfrac{x}{y} = x^2 + c$

30. (B) In ΔABC,

sinA + sinB + sinC

$\Rightarrow 2\sin\dfrac{A+B}{2}.\cos\dfrac{A-B}{2} + 2\sin\dfrac{C}{2}.\cos\dfrac{C}{2}$

$\Rightarrow 2\sin\dfrac{180-C}{2}.\cos\dfrac{A-B}{2} + 2\sin\dfrac{C}{2}.\cos\dfrac{C}{2}$

$\Rightarrow 2\cos\dfrac{C}{2}.\cos\dfrac{A-B}{2} + 2\sin\dfrac{180-(A+B)}{2}.\cos\dfrac{C}{2}$

$\Rightarrow 2\cos\dfrac{C}{2}\left[\cos\dfrac{A-B}{2} + \sin\left[90 - \dfrac{A+B}{2}\right]\right]$

$\Rightarrow 2\cos\dfrac{C}{2}\left[\cos\dfrac{A-B}{2} + \cos\dfrac{A+B}{2}\right]$

$\Rightarrow 2\cos\dfrac{C}{2} \times 2\cos\dfrac{\frac{A-B}{2}+\frac{A+B}{2}}{2}.\cos\dfrac{\frac{A-B}{2}+\frac{A+B}{2}}{2}$

$\Rightarrow 4\cos\dfrac{C}{2} \times \cos\dfrac{A}{2} \times \cos\left(\dfrac{-B}{2}\right)$

$\Rightarrow 4\cos\dfrac{A}{2}.\cos\dfrac{B}{2}.\cos\dfrac{C}{2}$

31. (C) $y = x^3 + e^x$

On differentiating both side w.r.t 'x'

$\dfrac{dy}{dx} = 3x^2 + e^x$

$\dfrac{dx}{dy} = \dfrac{1}{3x^2 + e^x}$

On differentiating both side w.r.t. 'y'

$\dfrac{d^2x}{dy^2} = -1.(3x^2 + e^x)^{-2}\,(6x + e^x)\,\dfrac{dx}{dy}$

$\dfrac{d^2x}{dy^2} = -\dfrac{6x + e^x}{(3x^2 + e^x)^2} \times \dfrac{1}{3x^2 + e^x}$

$\dfrac{d^2x}{dy^2} = -\dfrac{6x + e^x}{(3x^2 + e)^3}$

32. (C) n(S) = 6×6×6 = 216

$E = \begin{Bmatrix}(6,6,2),(6,5,3),(6,4,4),(6,3,5),(6,2,6)\\(5,6,3),(5,5,4),(5,4,5),(5,3,6),(4,6,4),\\(4,5,5),(4,4,6),(3,6,5),(3,5,6),(2,6,6)\end{Bmatrix}$

n(E) = 15

The required Probability P(E) = $\dfrac{n(E)}{n(S)}$

P(E) = $\dfrac{15}{216} = \dfrac{5}{72}$

33. (B) n = 10

Number of diagonals = $\dfrac{n(n-3)}{2}$

$= \dfrac{10 \times 7}{2} = 35$

34. (B) $\dfrac{\sin^2 3A}{\sin^2 A} - \dfrac{\cos^2 3A}{\cos^2 A}$

$\Rightarrow \left(\dfrac{\sin 3A}{\sin A}\right)^2 - \left(\dfrac{\cos 3A}{\cos A}\right)^2$

$\Rightarrow \left(\dfrac{3\sin A - 4\sin^3 A}{\sin A}\right)^2 - \left(\dfrac{4\cos^3 A - 3\cos A}{\cos A}\right)^2$

$\Rightarrow (3 - 4\sin^2 A)^2 - (4\cos^2 A - 3)^2$

$\Rightarrow 9 + 16\sin^4 A - 24\sin^2 A - 16\cos^4 A - 9 + 24\cos^2 A$

$\Rightarrow 16(\sin^4 A - \cos^4 A) - 24(\sin^2 A - \cos^2 A)$

$\Rightarrow (\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A) - 24(\sin^2 A - \cos^2 A)$

$\Rightarrow (\sin^2 A - \cos^2 A)[16(\sin^2 A + \cos^2 A) - 24]$

$\Rightarrow -(\cos^2 A - \sin^2 A)[16 - 24]$

$\Rightarrow 8\cos 2A$

35. (D) $I = \displaystyle\int_0^{\pi/2} \dfrac{\tan x - \cot x}{1 - \tan x.\cot x}\,dx$ ...(i)

$I = \displaystyle\int_0^{\pi/2} \dfrac{\tan\left(\frac{\pi}{2} - x\right) - \cot\left(\frac{\pi}{2} - x\right)}{1 - \tan\left(\frac{\pi}{2} - x\right).\cot\left(\frac{\pi}{2} - x\right)}\,dx$

$I = \displaystyle\int_0^{\pi/2} \dfrac{\cot x - \tan x}{1 - \tan x.\cot x}$ ...(ii)

from eq(i) and eq(ii)

$2I = 0 \Rightarrow I = 0$

36. (B) $I = \int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$

Let $\sin^{-1}x = t$ when $x \to 0,\ t \to 0$

$\frac{1}{\sqrt{1-x^2}}\, dx = dt$ $\quad x \to 1,\ t \to \frac{\pi}{2}$

$\Rightarrow I = \int_0^{\pi/2} t\, dx \Rightarrow I = \left[\frac{t^2}{2}\right]_0^{\pi/2}$

$\Rightarrow I = \frac{1}{2} \times \frac{\pi^2}{4} = \frac{\pi^2}{8}$

37. (D) $\lim_{x\to 3} \frac{4^{x/2} - 8}{2^{2x} - 64}$ $\quad \left[\frac{0}{0}\right]$ from

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 3} \frac{4^{x/2}(\log 4) \times \left(\frac{1}{2}\right) - 0}{2^{2x}(\log 2) \times (2) - 0}$

$\Rightarrow \lim_{x\to 3} \frac{2^x \times \frac{1}{2} \times 2\log 2}{2^{2x} \times 2\log 2} \Rightarrow \frac{1}{2} \times \frac{2^3}{2^6} = \frac{1}{16}$

38. (C) Straight line

$\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-1}{-2}$ and $\frac{x+1}{-2} = \frac{y-4}{4} = \frac{z+5}{5}$

Angle between the straight lines

$\cos\theta = \frac{3\times(-2) + 4\times 4 + (-2)\times 5}{\sqrt{3^2 + 4^2 + (-2)^2}\sqrt{(-2)^2 + 4^2 + 5^2}}$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90° = \frac{\pi}{2}$

39. (C) Determinant $\begin{vmatrix} 2 & 5 & 1 \\ 6 & 4 & 3 \\ 2 & -1 & 0 \end{vmatrix}$

Cofactor of 3 = $(-1)^{2+3}\begin{vmatrix} 2 & 5 \\ 2 & -1 \end{vmatrix}$

$= -1(-2 - 10) = 12$

40. (B) $\begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} -2 & -4 \\ 3 & -\lambda \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$

$\begin{bmatrix} -4+3 & -8-\lambda \\ -8+3 & -16-\lambda \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$

$\begin{bmatrix} -1 & -8-\lambda \\ -5 & -16-\lambda \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$

On camparing

$-8 - \lambda = -2 \Rightarrow \lambda = -6$

41. (D) $I = \int \frac{dx}{x(1+\log x)^3}$

Let $1 + \log x = t \Rightarrow \frac{1}{x}\, dx = dt$

$\Rightarrow I = \int \frac{dt}{t^3} \Rightarrow I = \frac{t^{-3+1}}{-3+1} + c$

$\Rightarrow I = \frac{-1}{2} \times \frac{1}{t^2} + c \Rightarrow I = \frac{-1}{2(1+\log x)^2} + c$

42. (B)

II	I
($\sin\theta$, $\mathrm{cosec}\theta$) → '+' other → '–'	All positive
($\tan\theta$, $\cot\theta$) → '+' other → '–'	($\cos\theta$, $\sec\theta$) → '+' other → '–'
III	IV

43. (C) 4 digit numbers formed from the digits 1, 2, 3, 4, 5, 6 , 7

| 7 | 7 | 7 | 7 | = 7 × 7 × 7 × 7 = 2401

44. (A) **Statement 1**

L.H.S. = $(\sec^2\theta - 1)(\mathrm{cosec}^2\theta - 1)$

$= \tan^2\theta.\cot^2\theta = 1$ = R.H.S.

Statement 1 is correct.

Statment 2

L.H.S. = $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}} + \frac{2\sin^2\frac{\theta}{2}\cdot 2\cos^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$

$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} + \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$= \frac{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}}{\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}} = \frac{1\times 2}{2\sin\frac{\theta}{2}\cdot\sin\frac{\theta}{2}}$

$= \frac{2}{\sin\theta} = 2\mathrm{cosec}\theta \neq$ R.H.S.

Statement 2 is incorrect.

45. (B) $ax^2 - \ x + c = 0$

Let roots = α and $\frac{1}{\alpha}$

Now, $\alpha \cdot \frac{1}{\alpha} = \frac{c}{a} \Rightarrow c = a$

46. (B) A' = cofactor of A

$|A'| = |\text{cofactor of A}|$

$|A'| = (A)^{4-1}$ [$\because$ Order = 4]

$|A'| = A^3$

47. (B) $y = \left(1 - x^{\frac{1}{8}}\right)\left(1 + x^{\frac{1}{4}}\right)\left(1 + x^{\frac{1}{2}}\right)\left(1 + x^{\frac{1}{8}}\right)$

$y = \left(1 + x^{\frac{1}{2}}\right)\left(1 + x^{\frac{1}{4}}\right)\left(1 - x^{\frac{1}{8}}\right)\left(1 + x^{\frac{1}{8}}\right)$

$y = \left(1 + x^{\frac{1}{2}}\right)\left(1 + x^{\frac{1}{4}}\right)\left(1^2 - \left(x^{\frac{1}{8}}\right)^2\right)$

$y = (1 + x^{1/2})(1 + x^{1/4})(1 - x^{1/4})$

$y = \left(1 + x^{\frac{1}{2}}\right)\left(1 - x^{\frac{1}{2}}\right)$

$y = 1 - x$

On differentiting both side w.r.t. 'x'

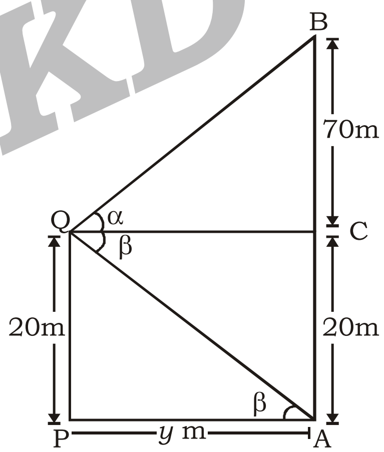
$\frac{dy}{dx} = -1$

48. (B) $\sin x \frac{dy}{dx} - y = x$

$\Rightarrow \frac{dy}{dx} - y \operatorname{cosec} x = x \operatorname{cosec} x$

$\Rightarrow \sin x \frac{dy}{dx} - y = x$

49. (C) **Case I:-**



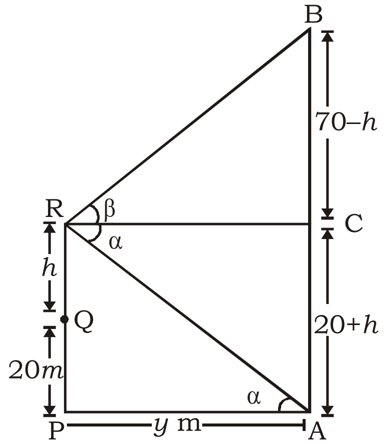
In ΔPQA:-

$\tan\beta = \frac{PQ}{PA} \Rightarrow \tan\beta = \frac{20}{PA}$ ...(i)

In ΔBCQ:-

$\tan\alpha = \frac{BC}{QC} \Rightarrow \tan\alpha = \frac{70}{PA}$ ...(ii)

Case II:-



Let he climps h m.

In ΔPAR:-

$\tan\alpha = \frac{PR}{PA} \Rightarrow \tan\alpha = \frac{20 + h}{PA}$ ...(iii)

In ΔBCR:-

$\tan\beta = \frac{BC}{RC} \Rightarrow \tan\beta = \frac{70 - h}{PA}$ ...(iv)

from eq(i) and eq(iv) or eq(ii) and eq(iii)

$\frac{20}{PA} = \frac{70 - h}{PA}$ or $\frac{70}{PA} = \frac{20 + h}{PA}$

$\boxed{h = 50\,m}$ $\boxed{h = 50\,m}$

50. (A) $[x^3 + 1] = (x + 1)(x^2 - x + 1)$

$[x^3 + 1] = (x + 1)(x + \omega)(x + \omega^2)$

51. (C) Area = $\int_0^1 \left(xe^{2x} - xe^{-2x}\right) dx$

Area = $\left[\left(x.\frac{e^{2x}}{2} + e^{2x}.1\right) - \left(x.\frac{e^{-2x}}{-2} + e^{-2x}.1\right)\right]_0^1$

Area = $\left[\frac{x}{2}e^{2x} + e^{2x} + \frac{x}{2}e^{-2x} - e^{-2x}\right]_0^1$

Area = $\left[\left(\frac{1}{2}e^2 + e^2 + \frac{1}{2}e^{-2} - e^{-2}\right) - (0 + 1 + 0 - 1)\right]$

Area = $\frac{3}{2}e^2 - \frac{1}{2}e^{-2}$

52. (D) $\{x : x + 6 = 6\} = \{0\}$

53. (C) $\frac{1 + \cos(B - C)\cos A}{1 + \cos(B - A)\cos C}$

$\Rightarrow \frac{1 - \cos(B - C)\cos(B + C)}{1 - \cos(B - A)\cos(B + A)}$ [$\because$ A+B+C =π]

$\Rightarrow \frac{1 - \cos^2 B + \sin^2 C}{1 - \cos^2 B + \sin^2 A}$

$\Rightarrow \frac{\sin^2 B + \sin^2 C}{\sin^2 B + \sin^2 A} = \frac{b^2 + c^2}{b^2 + a^2}$

54. (C) Given that

$x + y = 25$ ...(i)

A.T.Q.

$A = x^3y^2$

$\Rightarrow A = x^3(25 - x)^2$

$\Rightarrow A = 625x^3 + x^5 - 50x^4$

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{dA}{dx} = 1875x^2 + 5x^4 - 200x^3$ ...(i)

Again, differentiating

$\Rightarrow \frac{d^2A}{dx^2} = 3750x + 20x^3 - 600x^2$ ...(ii)

for maxima and minima

$\frac{dA}{dx} = 0$

$\Rightarrow 1875x^2 + 5x^4 - 200x^3 = 0$

$\Rightarrow 5x^2(x^2 - 40x + 375) = 0$

$\Rightarrow x^2(x - 25)(x - 15) = 0$

$\Rightarrow x = 0, 15, 25$

from eq. (ii)

$\left(\frac{d^2A}{dx^2}\right)_{\text{at } x=15} = 3750\times15+20\times15^3-600\times(15)^2$

$= -11250$ (maxima)

$\left(\frac{d^2A}{dx^2}\right)_{\text{at } x=25} = 3750\times25+20(25)^3-600\times(25)^2$

$= 31250$ (minima)

For maximum value, $x = 15$ and $y = 10$

55. (D) $\frac{dy}{dx} = 2xy - 2x + y - 1$

$\Rightarrow \frac{dy}{dx} = (y-1)(2x+1)$

$\Rightarrow \frac{dy}{y-1} = (2x+1)dx$

On differentiating

$\Rightarrow \log(y-1) = x^2 + x + \log c$

$\Rightarrow \log\left(\frac{y-1}{c}\right) = x^2 + x$

$\Rightarrow y - 1 = c.e^{x(x+1)} \Rightarrow y = 1 + c.e^{x(x+1)}$

56. (D) $y = 3^{\frac{1}{\log_x 9}} \Rightarrow y = 3^{\log_9 x} \Rightarrow y = 3^{\log_3\sqrt{x}}$

$\Rightarrow y = \sqrt{x} \Rightarrow x = y^2$

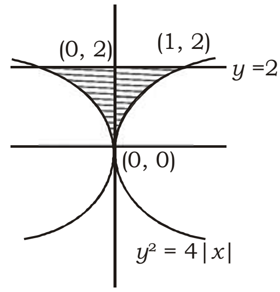
57. (C) $S = 3^2 + 6^2 + 9^2 + ... + 45^2$

$S = 3^2(1^2 + 2^2 + 3^2 + ... + 15^2)$

$S = 3^2 \times \frac{15}{6}(15+1)(2\times15+1)$

$S = 9 \times \frac{5}{2} \times 16 \times 31 = 11160$

58. (A)



Curve $x_1 \Rightarrow x = \frac{y^2}{4}$ and line $y = 2$

Area $= 2\int_0^2 x_1\, dy$

Area $= 2\int \frac{y^2}{4}\, dy$

Area $= 2 \times \left[\frac{y^3}{4\times3}\right]_0^2$

Area $= \frac{2}{12}[8 - 0] = \frac{4}{3}$ sq. unit

59. (B) $I = \int \sin^{-1}\left(\frac{1-x}{1+x}\right)dx$

Let $x = \tan^2\theta \Rightarrow dx = 2\tan\theta.\sec^2\theta.d\theta$

$I = \int \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) \times 2\tan\theta.\sec^2\theta.d\theta$

$I = \int \sin^{-1}(\cos 2\theta) \times 2\tan\theta.\sec^2\theta.d\theta$

$I = \int \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\} \times 2\tan\theta.\sec^2\theta.d\theta$

$I = \int 2\left(\frac{\pi}{2} - 2\theta\right)\sec^2\theta.\tan\theta d\theta$

$I = \pi\int \sec^2\theta\tan\theta\, d\theta - 4\int \theta.\sec^2\theta\tan\theta\, d\theta$

$I = \pi\frac{\tan^2\theta}{2} -$

$4\left[\theta.\int\tan\theta.\sec^2\theta.d\theta - \int\left\{\frac{d}{d\theta}(\theta).\int\tan\theta.\sec^2\theta\right\}dx\right]$

$I = \frac{\pi}{2}\tan^2\theta - 4\left[\theta.\frac{\tan^2\theta}{2} - \int 1.\frac{\tan^2\theta}{2}d\theta\right]$

$I = \frac{\pi}{2}\tan^2\theta - 2\theta.\tan^2\theta + 2\int(\sec^2\theta - 1)\,d\theta$

$I = \frac{\pi}{2}\tan^2\theta - 2\theta.\tan^2\theta + 2[\tan\theta - \theta] + c$

$I = \frac{\pi}{2}x - 2x.\tan^{-1}\sqrt{x} + 2[\sqrt{x} - \tan^{-1}\sqrt{x}] + c$

$I = \frac{\pi}{2}x - 2(x+1)\tan^{-1}\sqrt{x} + 2\sqrt{x} + c$

60. (A) $\begin{bmatrix} 1 & 0 & 1 \\ 3 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

61. (D) Given that $\tan A = \frac{-1}{3}$ and $\tan B = \frac{1}{2}$

Now, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A.\tan B}$

$\Rightarrow \tan(A - B) = \frac{\frac{-1}{3} - \frac{1}{2}}{1 + \left(\frac{-1}{3}\right) \times \frac{1}{2}}$

$\Rightarrow \tan(A - B) = \frac{\frac{-5}{6}}{\frac{5}{6}} \Rightarrow \tan(A - B) = -1$

$\Rightarrow A - B = \frac{3\pi}{4}$

62. (B) Ratio of angles = 2 : 2 : 1

Let angles = $2x$, $2x$, x

Now, $2x + 2x + x = 180$

$\Rightarrow 5x = 180 \Rightarrow x = 36$

Angel A = 72, B = 72, C = 36

Now, $\sin^2 A + \sin^2 B + \sin^2 C$

$\Rightarrow \sin^2 72 + \sin^2 72 + \sin^2 36$

$\Rightarrow \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 + \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 + \left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)^2$

$\Rightarrow \frac{10+2\sqrt{5}}{16} + \frac{10+2\sqrt{5}}{16} + \frac{10-2\sqrt{5}}{16}$

$\Rightarrow \frac{30+2\sqrt{5}}{16} = \frac{15+\sqrt{5}}{8}$

63. (C) $y = a^x \log_a a^x \Rightarrow y = a^{x \times x} \log_a a$

$\Rightarrow y = a^{x^2}$

On differentiating both side w. r. t. 'x'

$\Rightarrow \frac{dy}{dx} = a^{x^2} \log a \times 2x$

$\Rightarrow \frac{dy}{dx} = 2x.a^{x^2} \log a$

64. (A)

65. (C) 1 1 0 0 1   0. 1 0 1

$1 \times 2^0 = 1$ $\frac{1}{2} = 1 \times 2^{-1}$

$0 \times 2^1 = 0$ $0 = 0 \times 2^{-2}$

$0 \times 2^2 = 0$ $\frac{1}{8} = 1 \times 2^{-3}$

$1 \times 2^3 = 8$ $\frac{1}{2} + \frac{1}{8} = \frac{5}{8} = 0.625$

$1 \times 2^4 = \underline{16}$

25

Hence $(11\ 001.101)_2 = (25.625)_{10}$

66. (B) Given that $\alpha = 20^\circ$

Now, $\sin\alpha.\sin2\alpha.\sin4\alpha$

$\Rightarrow \sin20^\circ.\ \sin40^\circ.\ \sin80^\circ$

$\Rightarrow \frac{1}{2} \sin20\ [2\sin40.\sin80]$

$\Rightarrow \frac{1}{2} \sin20\ [\cos(40-80) - \cos(40+80)]$

$\Rightarrow \frac{1}{2} \sin20 \left[\cos 40 + \frac{1}{2}\right]$

$\Rightarrow \frac{1}{2} \sin20.\cos40 + \frac{1}{4} \sin20$

$\Rightarrow \frac{1}{4} \times 2\sin20.\ \cos40 + \frac{1}{4} \sin20$

$\Rightarrow \frac{1}{4}\left[\sin(20+40) + \sin(20-40)\right] + \frac{1}{4}\sin20$

$\Rightarrow \frac{1}{4}\left[\frac{\sqrt{3}}{2} - \sin 20\right] + \frac{1}{4} \sin20$

$\Rightarrow \frac{\sqrt{3}}{8} - \frac{1}{4} \sin20 + \frac{1}{4} \sin20 = \frac{\sqrt{3}}{8}$

67. (C) Equation $ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} \Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} \Rightarrow \frac{\left(-\frac{b}{a}\right)^2 - 2 \times \frac{c}{a}}{\frac{c}{a}}$

$\Rightarrow \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}} \Rightarrow \frac{\frac{b^2 - 2ac}{a^2}}{\frac{c}{a}} = \frac{b^2 - 2ac}{ca}$

68. (B) In the expansion of $\left(\frac{2x}{y} - \frac{y}{6x}\right)^6$

Total term = 6 + 1 = 7

Middle term = $\left(\frac{6}{2} + 1\right)^{th} = 4^{th}$

$T_4 = T_{3+1} = {}^6C_3 \left(\frac{2x}{y}\right)^3 \left(\frac{-y}{6x}\right)^3$

$T_4 = 20 \times \frac{8x^3}{y^3} \left(\frac{-y^3}{216x^3}\right) = \frac{-20}{27}$

69. (A)

70. (C) Digits 0, 1, 2, 4, 5, 7, 9
when last digit is '0'

4	1	= 4

↓ 0

when last digit is '2'

4	1	= 4

↓ 2

when last digit is '4'

3	1	= 3

↓ 4

The required numbers = 4 + 4 + 3 = 11

71. (B) $n(S) = 16$
E = {(HHHT), (HTHH), (HHTH), (THHH)}
$n(E) = 4$

The required Probability $= \frac{n(E)}{n(S)} = \frac{4}{16} = \frac{1}{4}$

72. (C) tan390 – cot690
$\Rightarrow \tan(360 + 30) - \cot(720 - 30)$

$\Rightarrow \tan 30° + \cot 30 \Rightarrow \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{4}{\sqrt{3}}$

73. (D) $\frac{\sin 330°.\cot 75°.\tan 135°}{\cos 425°.\sin 750°.\cot 225°}$

$\Rightarrow \frac{\sin(360-30).\cos 75.\tan(90+45)}{\cos(360+75).\sin(720+30).\cot(180+45)}$

$\Rightarrow \frac{-\sin 30.\cos 75.(-\tan 45)}{\cos 75.\sin 30.\cot 45}$

$\Rightarrow \frac{-\frac{1}{2} \times \cos 75 \times (-1)}{\cos 75 \times \frac{1}{2} \times 1} = 1$

74. (C) $\lim_{x \to 0} \frac{25^x - 16^x}{x(5^x + 4^x)}$ $\left[\frac{0}{0}\right]$form
by L-Hospital's Rule

$\Rightarrow \lim_{x \to 0} \frac{25^x \log 25 - 16^x \log 16}{x(5^x \log 5 + 4^x \log 4) + (5^x + 4^x)}$

$\Rightarrow \frac{25^0 \times 2\log 5 - 16^0 \times 4 \log 2}{0 + (5^0 + 4^0)}$

$\Rightarrow \frac{2\log 5 - 4\log 2}{2} \Rightarrow \log 5 - 2\log 2$

$\Rightarrow \log 5 - \log 4 = \log \frac{5}{4}$

75. (A) $\sin^{-1}x = \cot^{-1}y$

$\Rightarrow \sin^{-1}x = \tan^{-1}\frac{1}{y}$ $\left[\because \tan^{-1} A = \cot^{-1}\frac{1}{A}\right]$

$\Rightarrow \sin^{-1}x = \sin^{-1}\frac{\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}$ $\left[\because \tan^{-1} A = \sin^{-1}\frac{A}{\sqrt{1+A^2}}\right]$

$\Rightarrow \sin^{-1}x = \sin^{-1}\frac{1}{\sqrt{y^2+1}}$

$\Rightarrow x = \frac{1}{\sqrt{y^2+1}} \Rightarrow x^2 = \frac{1}{y^2+1}$

$\Rightarrow x^2(1 + y^2) = 1$

76. (C) Let the equation of sphere
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ ...(i)
its passes through the points (0, 0, 0), (–1, 0, 0), (0, –3, 0) and (0, 0, 4)
$d = 0$ ...(ii)

$1 + 2u(-1) + d = 0 \Rightarrow u = \frac{1}{2}$ ...(iii)

$9 + 2v(-3) + d = 0 \Rightarrow v = \frac{3}{2}$ ...(iv)

$16 + 2w(4) + d \Rightarrow w = -2$ ...(v)

On putting the value of u, u, w and d in eq(i)

$\Rightarrow x^2 + y^2 + z^2 + 2 \times \frac{1}{2}x + 2 \times \frac{3}{2}y + 2 \times (-2)z = 0$

$\Rightarrow x^2 + y^2 + z^2 + x + 3y - 4z = 0$

77. (B) $I = \int_0^{\pi/2} \frac{(\sin x)^{3/2}}{(\sin x)^{3/2} + (\cos x)^{3/2}}$ ...(i)

Prop.IV $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$I = \int_0^{\pi/2} \frac{\left[\sin\left(\frac{\pi}{2} - x\right)\right]^{3/2}}{\left[\sin\left(\frac{\pi}{2} - x\right)\right]^{3/2} + \left[\cos\left(\frac{\pi}{2} - x\right)\right]^{3/2}} dx$

$I = \int_0^{\pi/2} \frac{(\cos x)^{3/2}}{(\cos x)^{3/2} + (\sin x)^{3/2}} dx$ ...(ii)

from eq(i) and eq(ii)

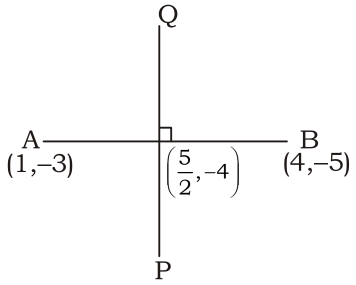
$2I = \int_0^{\pi/2} \frac{(\sin x)^{3/2} + (\cos x)^{3/2}}{(\cos x)^{3/2} + (\sin x)^{3/2}} dx$

$2I = \int_0^{\pi/2} 1.dx$

$2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$

78. (A)



Mid = point of joining the points =

$\left(\frac{1+4}{2},\frac{-3-5}{2}\right) = \left(\frac{5}{2},-4\right)$

Slope of line AB (m_1) = $\frac{-5+3}{4-1} = \frac{-2}{3}$

Slope of line PQ (m_2) = $\frac{-1}{-2/3} = \frac{3}{2}$

Equation of line PQ

$y + 4 = \frac{3}{2}\left(x-\frac{5}{2}\right) \Rightarrow y + 4 = \frac{3}{2}\times\frac{2x-5}{2}$

$\Rightarrow 6x - 4y = 31$

79. (C) $\left[\frac{\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}}\right]^2$

$\Rightarrow \frac{\cos\left(2\times\frac{\pi}{4}\right)+i\sin\left(2\times\frac{\pi}{4}\right)}{\cos\left(2\times\frac{\pi}{4}\right)-i\sin\left(2\times\frac{\pi}{4}\right)}$

$\Rightarrow \frac{\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}} \Rightarrow \frac{0+i\times1}{0-i\times1} = \frac{i}{-i} = -1$

80. (A) $(A\cap C)\cup(B\cap C)$

81. (B) $m = \begin{vmatrix}-1 & -1\\ 1 & -1\end{vmatrix} = 1 + 1 = 2$

$n = \begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix} = -1 - 1 = -2$

Now, $m\sin^2\theta - n\cos^2\theta$

$\Rightarrow 2\sin^2\theta + 2\cos^2\theta = 2$

82. (B) Equation whose roots are –7 and –6, then $(x + 7)(x + 6) = 0$

$\Rightarrow x^2 + 13x + 42 = 0$

Original equation

$x^2 + 17x + 42 = 0$

$\Rightarrow (x + 14)(x + 3) = 0$

Hence roots are –14 and –3.

83. (D) $A = \begin{bmatrix}3 & 1\\ 2 & -4\end{bmatrix}$

$A^2 = \begin{bmatrix}3 & 1\\ 2 & -4\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & -4\end{bmatrix}$

$A^2 = \begin{bmatrix}11 & -1\\ -2 & 18\end{bmatrix}$

Now, $A^2 + A - 14I = \begin{bmatrix}11 & -1\\ -2 & 18\end{bmatrix} + -14\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow A^2 + A - 14I = \begin{bmatrix}14 & 0\\ 0 & 14\end{bmatrix} - \begin{bmatrix}14 & 0\\ 0 & 14\end{bmatrix}$

$\Rightarrow A^2 + A - 14I = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$\Rightarrow A^2 + A - 14I = 0$

84. (C) $\int_0^2\{k^2+(2+k)x+3x^2\}\,dx \le 36$

$\Rightarrow \left[k^2x+(2+k)\frac{x^2}{2}+3\times\frac{x^3}{3}\right]_0^2 \le 36$

$\Rightarrow 2k^2 + (2 + k)\times2 + 8 \le 36$

$\Rightarrow 2k^2 + 2k - 24 \le 0$

$\Rightarrow (2k - 6)(k + 4) \le 0$

$\Rightarrow (k - 3)(k + 4) \le 0$

Hence $-4 \le k \le 3$

85. (C) $f(x) = \frac{1}{\sqrt{29-x^2}} \Rightarrow f'(x) = \frac{x}{(29-x^2)^{3/2}}$

Now, $\lim_{x\to2}\frac{f(2)-f(x)}{x^3-8}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to2}\frac{-f'(x)}{3x^2} \Rightarrow \lim_{x\to2}\frac{\frac{-x}{(29-x^2)^{3/2}}}{3x^2}$

$\Rightarrow \frac{\frac{-2}{(29-4)^{3/2}}}{3\times4} \Rightarrow \frac{-2}{12\times125} = \frac{-1}{750}$

86. (B) Series $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\ldots+\frac{1}{34\times37}$

$\Rightarrow \frac{1}{3}\left[\left(1-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\ldots+\left(\frac{1}{34}-\frac{1}{37}\right)\right]$

$\Rightarrow \frac{1}{3}\left[1-\frac{1}{37}\right] \Rightarrow \frac{1}{3}\times\frac{36}{37} = \frac{12}{37}$

87. (A) $\overline{A}\cap B\cap C$

88. (C) $n(S) = {}^{14}C_4 = 1001$

$n(E) = {}^6C_3 \times {}^3C_1 \times {}^5C_0 + {}^6C_3 \times {}^3C_0 \times {}^5C_1 + {}^6C_4 \times {}^3C_0 \times {}^5C_0$

$n(E) = 20 \times 3 \times 1 + 20 \times 1 \times 5 + 15 \times 1 \times 1$

$n(E) = 60 + 100 + 15 = 175$

The required Probability $= \frac{175}{1001} = \frac{25}{143}$

89. (A) $\begin{vmatrix} 0 & a & b \\ b & 0 & a \\ a & b & 0 \end{vmatrix} = 0 \Rightarrow -a(0 - a^2) + b(b^2 - 0) = 0$

$\Rightarrow a^3 + b^3 = 0$

90. (C) The required number of ways $= {}^{15-1}C_{11-1}$

$= {}^{14}C_{10} = 1001$

91. (D) Given that $f(x) = \frac{x-1}{x+1}$

Now, $\frac{f(x)+1}{f(x)-1} + x \Rightarrow \frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1} + x$

$\Rightarrow \frac{x-1+x+1}{x-1-x-1} + x \Rightarrow \frac{2x}{-2} + x = 0$

92. (C) $f(f(x)) = f[f(x)]$

$\Rightarrow f(f(x)) = f\left[\frac{x-1}{x+1}\right]$

$\Rightarrow f(f(x)) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}$

$\Rightarrow f(f(x)) = \frac{x-1-x-1}{x-1+x+1}$

$\Rightarrow f(f(x)) = \frac{-2}{2x} = \frac{-1}{x}$

93. (A) A.T.Q,

$\frac{\text{AM}}{\text{G.M}} = \frac{5}{4} \Rightarrow \frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{5}{4}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$

by Componedo & Dividendo Rule

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+4}{5-4} \Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{9}{1}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{3}{1}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{3+1}{3-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{4}{2} \Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{2}{1}$

On squaring

$\Rightarrow \frac{a}{b} = \frac{4}{1}$

Hence $= a : b = 4 : 1$

94. (C) Given that $g(x) = x$, $f(x) = \frac{1}{g(x)} = \frac{1}{x}$

L.H.S. $= f(g(g(f(x)))) = f\left(g\left(g\left(\frac{1}{x}\right)\right)\right)$

$= f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{x}\right) = x$

R.H.S $= g(f(f(g(x)))) = g(f(f(x)))$

$= g\left(f\left(\frac{1}{x}\right)\right) = g(x) = x$

L.H.S = R.H.S

Hence option (C) is correct.

95. (A)

2	37	1
2	18	0
2	9	1
2	4	0
2	2	0
2	1	1
	0	

Hence $(37)_{10} = (100101)_2$

96. (B) B is a 2×3 matrix.

97. (C) The required no. of triangles $= {}^{14}C_3 - {}^8C_3$

$= 364 - 56 = 308$

98. (D) $y = \sqrt{e^{\sqrt{x}}}$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = \frac{1}{2}\left(e^{\sqrt{x}}\right)^{-1/2} \times e^{\sqrt{x}} \times \frac{1}{2}(x)^{-1/2}$

$\frac{dy}{dx} = \frac{1}{4} \times \frac{e^{\sqrt{x}}}{\sqrt{x}.\sqrt{e^{\sqrt{x}}}}$

and $z = e^x \Rightarrow \frac{dz}{dx} = e^x$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\Rightarrow \frac{dy}{dz} = \frac{1}{4} \times \frac{e^{\sqrt{x}}}{\sqrt{x}.\sqrt{e^{\sqrt{x}}}} \times \frac{1}{e^x}$

$\Rightarrow \frac{dy}{dz} = \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}.e^x}$

99. (B) $I = \int \frac{3 - 2\sin x}{\cos^2 x}\, dx$

$I = \int \left(3\sec^2 x - 2\sec x.\tan x\right) dx$

$I = 3\tan x - 2\sec x + c$

100. (A) Equation $ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$

$= \frac{c}{a}\left(\frac{-b}{a}\right) = \frac{-bc}{a^2}$

and $\alpha^2\beta.\alpha\beta^2 = (\alpha\beta)^3 = \left(\frac{c}{a}\right)^3 = \frac{c^3}{a^3}$

The required equation

$x^2 - (\alpha^2\beta + \alpha\beta^2)x + \alpha^2\beta.\alpha\beta^2 = 0$

$\Rightarrow x^2 + \frac{bc}{a^2}x + \frac{c^3}{a^3} = 0$

$\Rightarrow a^3 + abcx + c^3 = 0$

101. (B) Line $\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z-1}{5}$

Direction cosines =

$\left\langle \frac{3}{\sqrt{3^2+(-4)^2+5^2}}, \frac{-4}{\sqrt{3^2+(-4)^2+5^2}}, \frac{5}{\sqrt{3^2+(-4)^2+5^2}} \right\rangle$

$= \left\langle \frac{3}{5\sqrt{2}}, \frac{-4}{5\sqrt{2}}, \frac{5}{5\sqrt{2}} \right\rangle = \left\langle \frac{3}{5\sqrt{2}}, \frac{-2\sqrt{2}}{5}, \frac{1}{\sqrt{2}} \right\rangle$

102. (B) Differential equation

$x.\log x \frac{dy}{dx} + y = \frac{2}{x}\log x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = \frac{2}{x^2}$

here, $P = \frac{1}{x\log x}$ $Q = \frac{2}{x^2}$

I. F. $= e^{\int P\,dx}$

I. F. $= e^{\int \frac{1}{x\log x}dx}$

We know that $\int \frac{1}{x\log x}dx = \log(\log x) + c$

I. F. $= e^{\log(\log x)} = \log x$

Solution of the differential equation

$y \times$ I. F. $= \int Q \times \text{I.F. } dx$

$\Rightarrow y \times \log x = \int \frac{2}{x^2} \times \log x\ dx$

Let $\log x = t \Rightarrow x = e^t \Rightarrow \frac{1}{x}dx = dt$

$\Rightarrow y\log x = 2\int t.e^{-t}\, dt$

$\Rightarrow y\log x = 2\left[t\int e^{-t}dt - \int\left\{\frac{d}{dt}(t).\int e^{-t}dt\right\}dt\right]$

$\Rightarrow y\log x = 2\left[-t.e^{-t} - \int 1.\left(-e^{-t}\right)dt\right]$

$\Rightarrow y\log x = 2\left[-t.e^{-t} - e^{-t}\right] + c$

$\Rightarrow y\log x = 2\left[-(\log x).\frac{1}{x} - \frac{1}{x}\right] + c$

$\Rightarrow y\log x = -2\left[\frac{1+\log x}{x}\right] + c$

103. (C) In the expansion of $\left(2\sqrt{x} - \frac{1}{2\sqrt{x}}\right)^8$

Middle term $= \left(\frac{8}{2}+1\right)^{th} = 5^{th}$

$T_5 = T_{4+1} = {}^8C_4 \left(2\sqrt{x}\right)^4\left(\frac{-1}{2\sqrt{x}}\right)^4$

$= 70 \times 1 = 70$

104. (B) $\vec{a} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

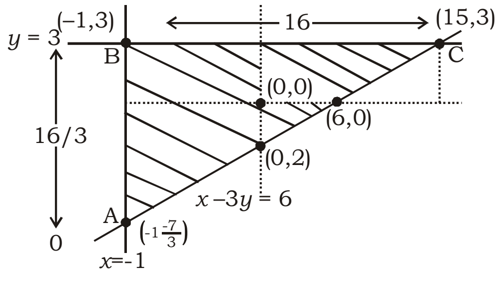
Projection of $\vec{a}$ on $\vec{b} = \left|\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right|$

$= \left|\frac{3\times 2 - 4\times 1 + 5\times(-2)}{\sqrt{2^2+1^2+(-2)^2}}\right|$

$= \left|\frac{6-4-10)}{\sqrt{9}}\right| = \frac{8}{3}$

105. (B)

106. (C)



Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

Area of $\Delta ABC = \frac{1}{2} \times \frac{16}{3} \times 16 = \frac{128}{3}$ sq. unit

107. (C) $y = 1 + \left(\frac{x}{3}\right) + \left(\frac{x}{3}\right)^2 + \left(\frac{x}{3}\right)^3 + \ldots\ldots\ldots\ldots$

$\Rightarrow y = \frac{1}{1-\frac{x}{3}} \Rightarrow 1 - \frac{x}{3} = \frac{1}{y}$

$\Rightarrow \frac{x}{3} = 1 - \frac{1}{y} \Rightarrow x = 3\left(1 - \frac{1}{y}\right)$

108. (B) P(26, 18) = k.C(26, 8)

$\Rightarrow \frac{26!}{(26-18)!} = k.\frac{26!}{8!(26-8)!}$

$\Rightarrow \frac{1}{8!} = k \times \frac{1}{8! \times 18!} \Rightarrow k = 18!$

109. (B) A.T.Q.

$2a = 3 \times 2b$

$\Rightarrow a = 3b$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 9b^2(1 - e^2)$

$\Rightarrow \frac{1}{9} = 1 - e^2$

$\Rightarrow e^2 = \frac{8}{9} \Rightarrow e = \frac{2\sqrt{2}}{3}$

110. (A) Class size = 14 – 11.5 = 2.5

111. (B) $\lim_{n\to\infty} \frac{1^2+2^2+3^2+......+n^2}{1^3+2^3+3^3+......+n^3}$

$\Rightarrow \lim_{n\to\infty} \frac{\frac{n}{6}(n+1)(2n+1)}{\frac{n^2(n+1)^2}{4}} \Rightarrow \lim_{n\to\infty} \frac{2(2n+1)}{3n(n+1)}$

$\Rightarrow \lim_{n\to\infty} \frac{2 \times n\left(2+\frac{1}{n}\right)}{3n^2\left(1+\frac{1}{n}\right)} \Rightarrow \lim_{n\to\infty} \frac{2\left(2+\frac{1}{n}\right)}{3n\left(1+\frac{1}{n}\right)}$

$\Rightarrow \frac{1}{\infty} = 0$

112. (C) Differential quation

$\sin\left(\frac{dy}{dx}\right) = x \Rightarrow \frac{dy}{dx} = \sin^{-1}x$

$\Rightarrow dy = \sin^{-1}x\, dx$

On integrating

$\Rightarrow \int dy = \int \sin^{-1}x\ dx$

$y = \sin^{-1}x. \int 1dx - \int\left\{\frac{d}{dx}(\sin^{-1}x).\int 1.dx\right\} dx$

$y = (\sin^{-1}x).\ x - \int \frac{1}{\sqrt{1-x^2}} \times x\, dx$

$y = x.\ \sin^{-1}x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\ dx$

$y = x.\ \sin^{-1}x + \frac{1}{2} \times \frac{(1-x^2)^{\frac{-1}{2}+1}}{-\frac{1}{2}+1} + c$

$y = x.\ \sin^{-1}x + \sqrt{1-x^2} + c$

113. (C) $\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{BA} = 3\hat{i} - \hat{j} + 2\hat{k}$

Area of ΔABC = $\frac{1}{2}\left|\overrightarrow{BC} \times \overrightarrow{BA}\right|$

Area of ΔABC = $\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -1 & 2\end{matrix}\right\|$

Area of ΔABC = $\frac{1}{2}\left\|\left[-\hat{i} - \hat{j} + \hat{k}\right]\right\|$

Area of ΔABC = $\frac{1}{2}\sqrt{(-1)^2 + (-1)^2 + 1^2} = \frac{\sqrt{3}}{2}$

114. (B) $f(x) = \frac{x^3 - 7x + 6}{x^3 - 2x^2 - 5x + 6}$

$f(x) = \frac{(x+3)(x-2)(x-1)}{(x-3)(x+2)(x-1)}$

$f(x) = \frac{(x+3)(x-2)}{(x-3)(x+2)} \Rightarrow f(1) = \frac{2}{3}$

$f'(x) = \frac{(x-3)(x+2)(2x+1) - (x+3)(x-2)(2x-1)}{(x-3)^2(x+2)^2}$

$f'(x) = \frac{(x^2-x-6)(2x+1) - (x^2+x-6)(2x-1)}{(x-3)^2(x+2)^2}$

$f'(x) = \frac{-4x^2 + 2x^2 - 12}{(x-3)^2(x+2)^2}$

$f'(x) = \frac{-2x^2 - 12}{(x-3)^2(x+2)^2}$

$f'(1) = \frac{-2-12}{(-2)^2(3)^2} = \frac{-14}{4 \times 9} = \frac{-7}{18}$

Now, $f(1) + f'(1) = \frac{2}{3} - \frac{7}{18}$

$\Rightarrow f(1) + f'(1) = \frac{12-7}{18} = \frac{5}{18}$

115. (B) A.T.Q,

$a + (p-1)d = \frac{1}{q}$ ...(i)

$a + (q-1)d = \frac{1}{p}$ ...(ii)

from eq(i) and eq(ii)

$a = \dfrac{1}{pq}$ and $d = \dfrac{1}{pq}$

Now, $T_{pq} = a + (pq - 1)d$

$\Rightarrow T_{pq} = \dfrac{1}{pq} + \dfrac{pq-1}{pq}$

$\Rightarrow T_{pq} = \dfrac{1+pq-1}{pq} = 1$

116. (D) $I = \int_{-\pi}^{\pi} |\cos x|\, dx$

$I = 2\int_{0}^{\pi} |\cos x|\, dx$

$I = 2\times 2\int_{0}^{\pi/2} \cos x\, dx$

$I = 4[\sin x]_{0}^{\pi/2}$

$I = 4\left[\sin\dfrac{\pi}{2} - \sin 0\right]$

$I = 4[1 - 0] = 4$

117. (B) $(1 - \sec A + \tan A)^2$

$\Rightarrow 1 + \sec^2 A + \tan^2 A - 2\sec A - 2\sec A.\tan A + 2\tan A$

$\Rightarrow \sec^2 A + 1 + \tan^2 A - 2\sec A - 2\sec A.\tan A + 2\tan A$

$\Rightarrow \sec^2 A + \sec^2 A - 2\sec A - 2\sec A.\tan A + 2\tan A$

$\Rightarrow 2\sec^2 A - 2\sec A - 2\sec A.\tan A + 2\tan A$

$\Rightarrow 2\sec A(\sec A - 1) - 2\tan A(\sec A - 1)$

$\Rightarrow (\sec A - 1)(2\sec A - 2\tan A)$

$\Rightarrow 2(\sec A - 1)(\sec A - \tan A)$

118. (B)

119. (D) Plane $3x - 4y + z = 13$

from option (D) (5, 2, 6)

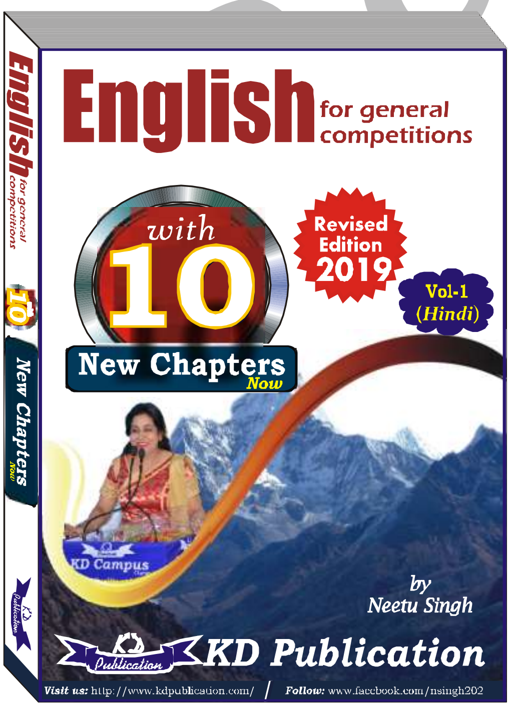
$3\times 5 - 4\times 2 + 6 = 13$

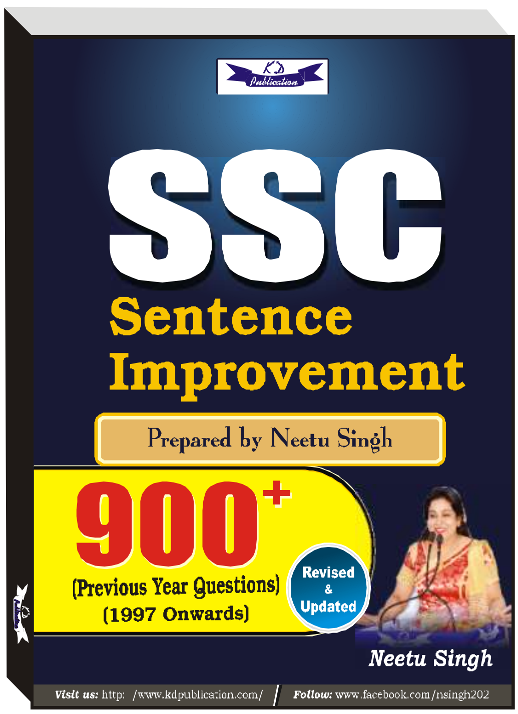
$\Rightarrow 15 - 8 + 6 = 13$

$\Rightarrow 13 = 13$

Hence option (D) is correct.

120. (D)





NDA (MATHS) MOCK TEST - 196 (Answer Key)

1.	(C)	21.	(B)	41.	(D)	61.	(D)	81.	(B)	101.	(B)
2.	(B)	22.	(D)	42.	(B)	62.	(B)	82.	(B)	102.	(B)
3.	(C)	23.	(B)	43.	(C)	63.	(C)	83.	(D)	103.	(C)
4.	(C)	24.	(B)	44.	(A)	64.	(A)	84.	(C)	104.	(B)
5.	(C)	25.	(A)	45.	(B)	65.	(C)	85.	(C)	105.	(B)
6.	(A)	26.	(B)	46.	(B)	66.	(B)	86.	(B)	106.	(C)
7.	(B)	27.	(A)	47.	(B)	67.	(C)	87.	(A)	107.	(C)
8.	(A)	28.	(A)	48.	(B)	68.	(B)	88.	(C)	108.	(B)
9.	(B)	29.	(C)	49.	(C)	69.	(A)	89.	(A)	109.	(B)
10.	(B)	30.	(B)	50.	(A)	70.	(C)	90.	(C)	110.	(A)
11.	(A)	31.	(C)	51.	(C)	71.	(B)	91.	(D)	111.	(B)
12.	(A)	32.	(C)	52.	(D)	72.	(C)	92.	(C)	112.	(C)
13.	(A)	33.	(B)	53.	(C)	73.	(D)	93.	(A)	113.	(C)
14.	(A)	34.	(B)	54.	(C)	74.	(C)	94.	(C)	114.	(B)
15.	(A)	35.	(D)	55.	(D)	75.	(A)	95.	(A)	115.	(B)
16.	(A)	36.	(B)	56.	(D)	76.	(C)	96.	(B)	116.	(D)
17.	(A)	37.	(D)	57.	(C)	77.	(B)	97.	(C)	117.	(B)
18.	(A)	38.	(C)	58.	(A)	78.	(A)	98.	(D)	118.	(B)
19.	(A)	39.	(C)	59.	(B)	79.	(C)	99.	(B)	119.	(D)
20.	(B)	40.	(B)	60.	(A)	80.	(A)	100.	(A)	120.	(D)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777