QUANTITATIVE ABILITY - 68 (SOLUTION)

1. (B) Let the two integers be a and b.
A.T.Q,
a + b = 16 .... (i)
and, $\frac{1}{a}+\frac{1}{b} = \frac{1}{3}$
$\frac{a+b}{ab} = \frac{1}{3}$
ab = 16 × 3 = 48 .... (ii)
We know that
$(a – b)^2 = (a + b)^2 – 4ab$
On putting the respective values, we get
$(a – b)^2 = 16^2 – 4 × 48$
$(a – b)^2 = 256 – 192$
$(a – b)^2 = 64$
(a – b) = 8
Hence, the difference of the integers is 8.

2. (D) For being completely divisible, the numerator must have the factor of denominator
Here, 65 k = 5 × 13 × k
and, 122 = 2 × 61
There is no common factor
So, the minimum value of k = 122

3. (A) Here, $(5^{70} + 7^{70})$ can be written as
$(5^2)^{35} + (7^2)^{35}$ i.e, $(25^{35} + 49^{35})$
We know that, $x^n + y^n$ is always completely divisible by $x + y$. When n is an old number
So, $25^{35} + 49^{35}$ will be divisible by
25 + 49 = 74
∴ Required remainder = 0

4. (B) $\left(\frac{x^p}{x^q}\right)^{p^2+q^2+pq} \cdot \left(\frac{x^q}{x^r}\right)^{q^2+r^2+qr} \cdot \left(\frac{x^r}{x^p}\right)^{r^2+p^2+rp} = x^{(p-q)(p^2+q^2+pq)} \cdot x^{(q-r)(q^2+r^2+qr)} \cdot x^{(r-p)(r^2+p^2+rp)}$
$= x^{(p^3-q^3)} \cdot x^{(q^3-r^3)} \cdot x^{(r^3-p^3)} = x^{(p^3-q^3+q^3-r^3+r^3-p^3)}$
$= x^\circ = 1$

5. (C) We know that,
HCF of $(a^m – 1)$ and $(a^n – 1) = a^{\text{HCF of } m \text{ and } n}–1$
HCF of $(5^{15}– 1)$ and $(5^{35} – 1) = 5^{\text{HCF of 15 and 35}}–1 = 5^5 – 1$

6. (B) Let the two numbers be $5x$ and $5y$.
Then,
Their LCM, $5xy$ = 1105
xy = 221
xy = 13 × 17
A.T.Q,
$5x + 5y$ = 150
$x + y$ = 30
Here, we get x = 13 and y = 17
Now, the difference of the numbers = 5y – 5x
= 5(17 – 13) = 20

7. (C) 25% = $\frac{1 \longrightarrow \text{Profit}}{4 \longrightarrow \text{SP}}$

We know that,

CP = SP – Profit = 4 – 1 = 3

Now, profit percent = $\frac{\text{Profit}}{\text{CP}} \times 100$ = $\frac{1}{3} \times 100 = 33\frac{1}{3}$

8. (B) Total distance travelled = 30 + 60 + 90 + ... upto 10 terms

Here, first term (a) = 30

common difference (d) = 30

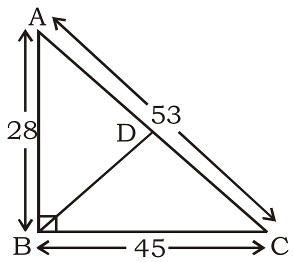
and, number of terms (n) = 10

Then, sum = $\frac{n}{2}[2a + (n-1)d]$

$= \frac{10}{2}[2 \times 30 + (10-1) \times 30] = 5[60 + 270] = 1650$

∴ Total distance travelled = 1650 metres.

9. (D)



Given triangle is right angle triangle with sides 28 cm, 45 cm and 53 cm.

Orthocentre of triangle ABC is B and circumcentre is the mid point of AC.

So,

Distance between orthocentre and circumcentre is equal to the length of BD where BD is the median and circumradius of the triangle.

∴ BD = $\frac{53}{2}$ = 26.5 cm

10. (C) Here, radius of the cone = half of the radius of semicircle = $\frac{28}{2}$ = 14 cm

and, slant height of the cone = radius of the circle

l = 28 cm

we know that,

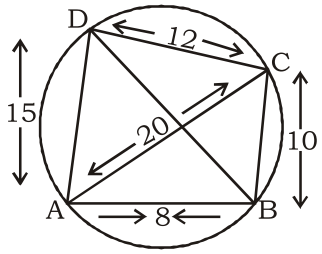
$h = \sqrt{l^2 - r^2}$

$h = \sqrt{28^2 - 14^2}$

$h = 14\sqrt{3}$ cm

∴ height of the cone = $14\sqrt{3}$ cm

11. (A)



In a quadrilateral ABCD,

AB × DC + BC × AD = AC × BD

On putting the values, we get

8 × 12 + 10 × 15 = 20 × BD

96 + 150 = 20 × BD

BD $= \frac{246}{20}$

BD = 12.3 cm

∴ Length of other diagonal = 12.3 cm

12. (B) Side of the square tiles = HCF of 75 and 100 = 25 *m*

and, number of tiles = $\frac{\text{Area of rectangular hall}}{\text{Area of one square hall}}$

$= \frac{75 \times 100}{25 \times 25} = 12$

∴ Minimum number of square tiles = 12

13. (D) Length of are AB = circumference of the base of right circular cone

$l = 2 \times \pi \times 3$

$l = 6\pi$

and, radius of the sector = slant height of cone

r = $\sqrt{4^2 + 3^2} = 5$

We know that,

angle subtended at the centre = $\frac{\text{length of arc}}{\text{radius}}$

$\theta = \frac{6\pi}{5}$

14. (C)

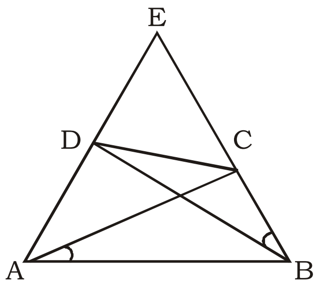
	A + B	A	B
Time →	x	$x+8$	$x + \frac{9}{2}$

Now, $x = \sqrt{8 \times \frac{9}{2}}$

$x = 6$

∴ Time taken by A and B together to complete the work = 6 days

15. (B)



Extenmd BC and AD to meet at E.

In ΔABE,

$\angle A = \angle B = 60°$

So, ΔABE is an equilateral.

In ΔABC and ΔBED (Given)

$\angle ABC = \angle BED$ (60°)

and, AB = BE (side of equilateral triangle)

So, $\Delta ABC \cong \Delta BED$

and, BC = DE

We know that,

AB = AE (side of equilateral triangle)

AB = AD + DE

AB = AD + BC

16. (C) In a triangle product of side and altitude remains same

Let the sides of the triangle be a, b and c.

Then,

$a \times 6 = b \times 7 = c \times 8$

Here, we get

$a : b : c = 28 : 24 : 21$

Now, (a + b + c) = (28 + 24 + 21) units

73 units = 365

1 unit = 5

Then, smallest side (c) = 5 × 21 = 105 cm

17. (A) We know that, the largest triangle that can be inscribed in a semicircle is an isosceles right angled triangle.

So, area = $\frac{1}{2} \times \left(\frac{\text{diameter}}{2}\right)^2$

$= \frac{1}{2} \times \left(\frac{2r}{2}\right)^2 = r^2$ sq. units

18. (C)

A	B	C
1000	950	950
1000	1000	940

1000 × 1000 : 950 ×1000 : 950 × 940

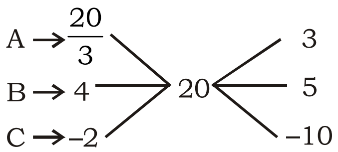
Now, Distance travelled by C when A travels 1000 m = $\frac{9540 \times 940}{1000}$ = 893 m

Then, Distance by which A can beat C = 1000 – 893 = 107 m

19. (B) We know that,

Volume of tetrahedron = $\frac{a^3}{6\sqrt{2}}$ = $\frac{6^3}{6\sqrt{2}}$ = $18\sqrt{2}$ cm³

20. (D)

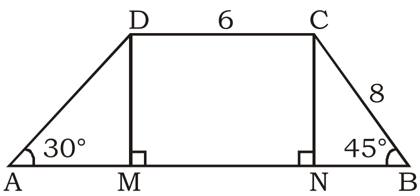


Now, work done by A, B and C in one hour = 3 + 5 – 10 = – 2 units

and, quantity of water in the half filled cistern = $\frac{20}{2}$ = 10 units

So, Time taken by all the pipes to empty the cistern = $\frac{10}{2}$ = 5 hours

21. (C)



In ΔCNB,

CN = CB sin45° = 8 × $\frac{1}{\sqrt{2}}$ = $4\sqrt{2}$ cm

and, BN = CB cos 45° = 8 × $\frac{1}{\sqrt{2}}$ = $4\sqrt{2}$ cm

Now, In ΔAMD,

DM = $4\sqrt{2}$ ($\because$ DM = CN)

Then, AM = DM cot30° = $4\sqrt{2} \times \sqrt{3}$ = $4\sqrt{6}$ cm

We know that,

Area of trapezium = $\frac{1}{2}$ × (sum of parallel sides × height) = $\frac{1}{2}$(DC + AB) × CN

= $\frac{1}{2}(6 + 4\sqrt{6} + 6 + 4\sqrt{2}) \times 4\sqrt{2}$ = $\frac{1}{2} \times (12 + 4\sqrt{6} + 4\sqrt{2}) \times 4\sqrt{2}$

= $8(2 + 2\sqrt{3} + 3\sqrt{2})$ = 85.65 cm²

22. (B) Let the radii of the two circles be R and r

Now,

$\pi R^2 + \pi r^2 = 125\pi$

$\pi (R^2 + r^2) = 125\pi$

$R^2 + r^2 = 125$ .... (i)

and, R – r = 5 .... (ii)

We know that,

$(a + b)^2 + (a – b)^2 = 2(a^2 + b^2)$

So, $(R + r)^2 + (R – r)^2 = 2(R^2 + r^2)$

$(R + r)^2 = 225$

R + r = 15 cm

23. (D) $x = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Rationalizing the denominator,

$x = 5 + 2\sqrt{6}$

and, $\dfrac{1}{x} = 5 - 2\sqrt{6}$

Then, $x + \dfrac{1}{x} = 5 + 2\sqrt{6} + 5 - 2\sqrt{6} = 10$

$x^2 - 10x + 1 = 0$ .... (i)

Multiply by x both sides, we get

$x^3 - 10x^2 + x = 0$ .... (ii)

Now, adding 4 × equation (i) and equation (ii),

$4x^2 - 40x + 4 + x^3 - 10x^2 + x = 0$

$x^3 - 6x^2 - 39x + 4 = 0$

$x^3 - 3x(2x + 13) = -4$

24. (C) A.T.Q,

$x^2 + \dfrac{1}{x^2} = \dfrac{7}{9}$

$\left(x + \dfrac{1}{x}\right)^2 = \dfrac{7}{9} + 2 = \dfrac{25}{9}$

$x + \dfrac{1}{x} = \dfrac{5}{3}$

On taking cube both sides, we get

$\left(x + \dfrac{1}{x}\right)^3 = \left(\dfrac{5}{3}\right)^3$

$x^3 + \dfrac{1}{x^3} + 3 \times x \times \dfrac{1}{x}\left(x + \dfrac{1}{x}\right) = \dfrac{125}{27}$

$x^3 + \dfrac{1}{x^3} + 3 \times \dfrac{5}{3} = \dfrac{125}{27}$

$x^3 + \dfrac{1}{x^3} = \dfrac{125}{27} - 5 = -\dfrac{10}{27}$

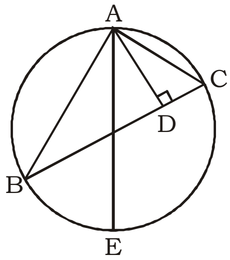
25. (C) In triangle ABC,

$|AB - AC| < |BC| < |AB + AC|$

$180 < BC < 1330$

Then, the number of possible number of triangles = 1330 – 180 – 1 = 1149

26. (B)



Circumradius of $\Delta ABC = \frac{abc}{4\Delta}$

where, b = AC = 12 cm

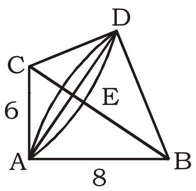
c = AB = 18 cm

a = BC

and AD = 6 cm

Now, circumradius (R) = $\frac{BC \times 12 \times 18}{4 \times \frac{1}{2} \times BC \times AD} = \frac{BC \times 12 \times 18}{4 \times \frac{1}{2} \times BC \times 6}$ = 18 cm

27. (C)



Volume of the double cone formed = volume of cone (ABD) + volume of cone (ACD)

$= \frac{1}{3}\pi (AE)^2 \times BE + \frac{1}{3}\pi (AE)^2 \times CE$

$= \frac{1}{3}\pi (AE)^2 (BE + CE) = \frac{1}{3}\pi (AE^2)(BC)$

Here, BC = $\sqrt{AB^2 + AC^2} = \sqrt{6^2 + 8^2}$ = 10 cm

and, AE = $\frac{AB \times AC}{BC} = \frac{8 \times 6}{10}$ = 4.8 cm

Now, required volume = $\frac{1}{3} \times 3.14 \times 4.8 \times 4.8 \times 10$

= 241.152 cm^3 = 240 cm^3 (approximate)

28. (B) $x = \sqrt{\frac{5 + 2\sqrt{6}}{5 - 2\sqrt{6}}}$

On rationalisation, we get

$x = \sqrt{\frac{(5 + 2\sqrt{6})(5 + 2\sqrt{6})}{(5 - 2\sqrt{6})(5 + 2\sqrt{6})}}$

$x = 5 + 2\sqrt{6}$

and, $\frac{1}{x} = 5 - 2\sqrt{6}$

Now, $x + \frac{1}{x} = 5 + 2\sqrt{6} + 5 - 2\sqrt{6}$

$x + \frac{1}{x} = 10$

$x^2 = 10x - 1$ .... (i)

Then, $x^2 - 7x - 14 = 10x - 1 - 7x - 14$

$= 3x - 15$

$= 3(5 + 2\sqrt{6}) - 15$

$= 15 + 6\sqrt{6} - 15 = 6\sqrt{6}$

29. (C) We know that, $\cot 2x = \frac{\cot^2 x - 1}{2\cot x} = \frac{1}{2}(\cot x - \tan x)$

Here, $\cot x = \frac{\sin y}{1 - \cos y} = \frac{1 + \cos y}{\sin y}$

and, $\tan x = \frac{1}{\cot x} = \frac{1 - \cos y}{\sin y}$

Then, $\cot 2x = \frac{1}{2}\left[\frac{(1 + \cos y) - (1 - \cos y)}{\sin y}\right] = \frac{1}{2} \times \frac{2\cos y}{\sin y} = \cot y$

30. (A) Rate of interest (r) = $\frac{SI \times 100}{p \times t}\%$

$r = \frac{16 \times 100}{100 \times 2} = 8\%$

Now, compound interest on ₹16000 in 3 years = $p\left[\left(1 + \frac{r}{100}\right)^3 - 1\right]$

$= 16000\left[\left(1 + \frac{8}{100}\right)^3 - 1\right] = ₹\, 4155$

31. (A) $a\sec\theta = x$

$\cos = \frac{a}{x}$ .... (i)

and, $b\tan\theta = y$

$\cos\theta = \frac{b\sin\theta}{y}$ .... (ii)

From equation (i) and (ii), we get

$\frac{a}{x} = \frac{b}{y}\sin\theta$

$\sin\theta = \frac{ay}{bx}$ .... (iii)

Using equation (i) and (iii),

$\sin^2\theta + \cos^2\theta = 1$

$\frac{a^2y^2}{b^2x^2} + \frac{a^2}{x^2} = 1$

$\frac{a^2y^2 + a^2b^2}{b^2x^2} = 1$

$b^2x^2 - a^2y^2 = a^2b^2$

32. (B) $\frac{210}{3\times7}+\frac{210}{7\times11}+\frac{210}{11\times15}+\ldots\ldots\frac{210}{31\times35}$

$=210\times\frac{1}{4}\left(\frac{1}{3}-\frac{1}{7}+\frac{1}{7}-\frac{1}{11}+\frac{1}{11}-\frac{1}{15}\ldots\ldots+\frac{1}{31}-\frac{1}{35}\right)=210\times\frac{1}{4}\left(\frac{1}{3}-\frac{1}{35}\right)$

$=210\times\frac{1}{4}\times\frac{32}{3\times35}=16$

33. (C) Let the side of the triangle be a cm

Then, A.T.Q,

$\frac{\sqrt{3}}{4}a^2-\frac{\sqrt{3}}{4}(a-2)^2=5\sqrt{3}$

$\frac{\sqrt{3}}{4}[a^2-(a-2)^2]=5\sqrt{3}$

[4a – 4] = 20

a = 6 cm

∴ Each side of equilateral triangle = 6 cm

34. (B) We know that,

$\text{Distance}=\frac{\text{Product of speeds}}{\text{Difference of speeds}}\times\text{time}$

$D=\frac{15\times20}{20-15}\times\frac{42}{60}$

D = 42 km

∴ Distance between his house and office = 42 km

35. (B) Total marks of 50 students of the class = 81 × 50 = 4050

and, total marks of 45 students of the class = 80 × 45 = 3600

Now, total marks obtained by 5 students = 4050 – 3600 = 450

Then, average marks of 5 students = $\frac{450}{5}=90$

36. (C) Let the marked price of the article be ₹x.

A.T.Q,

$x\times\frac{8-1}{8}\times\frac{80+7}{80}=3045$

$x\times\frac{7}{8}\times\frac{87}{80}=3045$

$x=\frac{3045\times8\times80}{7\times87}=3200$

∴ Marked price of the article = ₹3200

37. (B) Amount given by the man to his wife = $64200 \times \frac{25}{100}$ = ₹16,050

Then, amount given by the man to his sons = 64200 – 16050 = ₹48150

Now, $A\left[1+\frac{r}{100}\right]^3 = B\left[1+\frac{r}{100}\right]^5$

$\frac{A}{B} = \left(1+\frac{r}{100}\right)^2$

$\frac{A}{B} = \left(\frac{21}{20}\right)^2 = \frac{441}{400}$

i.e., the amount is distributed in A and B in the ratio = 441 : 400

Then, (441 + 400) units = 48150

841 units = 48150

1 unit = $\frac{48150}{841}$

Then, share of B = 400 units

$= \frac{48150}{841} \times 400$ = ₹22901.3

38. (D) SP of 60% = CP of 100%

100 60

Let the number of oranges be 100.

and, SP of the 60 oranges = ₹100

CP of the 60 oranges = ₹60

Then,

Now, profit percentage = $\frac{100-60}{60}\times 100 = \frac{200}{3}\%$

Again, SP of (100 – 60) × $\frac{60}{100}$ oranges = $24 \times \frac{4}{3} = 32$

Total CP = 100 and total SP = 100 + 32 = 132

Profit percentage = $\frac{32}{100} \times 100$ = 32%

39. (C) Let the speed of the car be x kmph

A.T.Q,

$\frac{400}{2000} \times 2 = \frac{x(x-10)}{10}\times 3$

$2x + 20 = 3x - 30$

$x = 50$

Then, distance between the two cities = $\frac{50(50+10)}{10}\times 2$ = 600 km

40 (B) $\text{cosec}\,\theta + \cot\theta = 2$

$$\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = 2$$

$$\frac{1+\cos\theta}{\sin\theta} = 2$$

$$\frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}} = 2$$

$$\cot\frac{\theta}{2} = 2$$

$$\tan\frac{\theta}{2} = \frac{1}{2}$$

We know that,

$$\sin\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$\sin\theta = \frac{2\times\frac{1}{2}}{1+\left(\frac{1}{2}\right)^2}$$

$$\sin\theta = \frac{1}{\frac{5}{4}} = \frac{4}{5}$$

41. (C) Area of the square when the wire is bent to form square = $\left(\frac{88}{4}\right)^2$ = 484 cm^2 when the wire is bent in the form of circle then radius of the circle = $\frac{88}{2\pi}$ = 14 cm

Then, area of the circle = πr^2 = $\frac{22}{7}\times14\times14 = 616\,cm^2$

Now, percentage change in the two enclosed areas = $\frac{616-484}{484}$ × 100 = 27.27%

42. (A) Let the CP of the goods be ₹100
Then, MP of the goods = ₹120
A.T.Q,

SP of half the stock = 120 × $\frac{1}{2}$ = ₹60

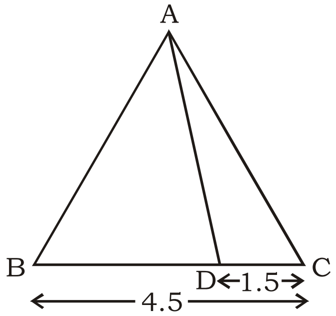
SP of $\frac{1}{4}$th of the stock = 60 × $\frac{1}{2}$ × $\frac{90}{100}$ = ₹ 27

and, SP of remaining $\frac{1}{4}$ stock = 30 × $\frac{80}{100}$ = ₹ 24

∴ Total SP of the good = 60 + 27 + 24 = ₹111

Now, profit percent = $\frac{111-100}{100}\times100$ = 11%

43. (A)



A.T.Q,
BD = BC – DC
BD = 4.5 – 1.5 = 3 cm
Now, BD : DC = 3 : 1.5 = 2 : 1

$\therefore$ Areas of ΔABD and ΔACD will be in the ratio 2 : 1

44. (C) $x = 5 + 2\sqrt{6}$

and, $\frac{1}{x} = 5 - 2\sqrt{6}$

Then, $x - \frac{1}{x} = (5 + 2\sqrt{6}) - (5 - 2\sqrt{6}) = 4\sqrt{6}$

Taking cube on both sides, we get

$x^3 - \frac{1}{x^3} - 3 \times x \times \frac{1}{x}\left(x - \frac{1}{x}\right) = \left(4\sqrt{6}\right)^3$

$x^3 - \frac{1}{x^3} - 3 \times 4\sqrt{6} = 384\sqrt{6}$

$x^3 - \frac{1}{x^3} = 396\sqrt{6}$

45. (B) $p(p^2 + 6p + 12) = p^3 + 6p^2 + 12p = (p + 2)^3 - 2^3$
$= (98 + 2)^3 - 8 = 100000 - 8 = 999992$

46. (C) We know that,

Distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-3)^2 + (8-2)^2} = \sqrt{2^2 + 6^2} = 2\sqrt{10}$ units

Then, radius of the circle = $\frac{2\sqrt{10}}{2} = \sqrt{10}$ units

Now, Area of circle = $\pi r^2 = \pi \times \sqrt{10} \times \sqrt{10} = 10\pi$ sq. units

47. (C) Required rate = $\frac{400}{2000} \times 100 = 20\%$

48. (A) Consider the equation $x^3 + qx + r = 0$
Here, sum of the roots = 0
i.e. a + b + c = 0

$\therefore$ $a^3 + b^3 + c^3 - 3abc = 0$

49. (A) We know that,

$$\frac{a}{\sin A} = \frac{b}{\sin B}\ \frac{c}{\sin C}$$

and, $a : b : c = 1 : \sqrt{3} : 2$

Here, $a^2 + b^2 = c^2$

So, $\angle C = 90°$

Now, $\frac{a}{\sin A} = \frac{c}{\sin c}$

$$\frac{1}{\sin A} = \frac{2}{\sin 90}$$

$\sin A = \frac{1}{2} \Rightarrow A = 30°$

and, $\frac{b}{\sin B} = \frac{c}{\sin c}$

$$\frac{\sqrt{3}}{\sin B} = \frac{2}{\sin 90}$$

$\sin B = \frac{\sqrt{3}}{2} = B = 60°$

$\therefore\ \angle A : \angle B : \angle C = 30° : 60° : 90° = 1 : 2 : 3$

50. (A) $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = \frac{\tan A}{1-\frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1-\tan A}$

$$= \frac{\tan^2 A}{\tan A - 1} - \frac{1}{\tan A(\tan A - 1)} = \frac{(\tan A - 1)(\tan^2 A + 1 + \tan A)}{\tan A(\tan A - 1)}$$

$= 1 + \tan A + \cot A$

51. (D) $x^2 - c^2 = y$

On putting the values, we get

$(a + b)^2 - c^2 = ab$

$a^2 + b^2 + 2ab - c^2 = ab$

$a^2 + b^2 - c^2 = -ab$

we know that

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

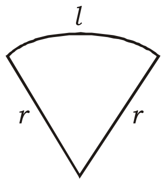
$$\cos C = \frac{-ab}{2ab}$$

$\cos C = \frac{-1}{2} \Rightarrow c = 120°$

Then, area of $\Delta ABC = \frac{1}{2} \times a \times b \times \sin 120°$

$= \frac{1}{2} \times a \times b = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} ab$

52. (C)



A.T.Q,

$r + r + l = 20$

$l = 20 - 2r$

We know that,

Area of sector = $\frac{1}{2} lr$

So, A = $\frac{1}{2} \times (20 - 2r) \times r$

A = $r(10 - r)$

For A to be maximum when r and (10 – r) are equal.

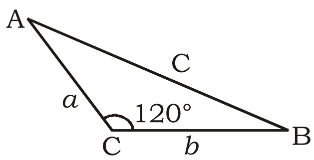
So, r = 10 – r

$2r = 10$

$r = 5$

Then, A = 5 × 5 = 25 sq. meter

53. (B)



Here, $a = 1 \times 5 = 5$ km,

and, $b = 3 \times 5 = 15$ km

We know that,

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\frac{-1}{2} = \frac{25 + 225 - c^2}{150}$

$c^2 = 325$

c = $5\sqrt{13}$ km

54. (A) $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$

$\cos\beta\cos\gamma + \sin\beta\sin\gamma + \cos\gamma\cos\alpha + \sin\gamma\sin\alpha + \cos\alpha\cos\beta + \sin\alpha\sin\beta = -\frac{3}{2}$

$3 + 2\cos\beta\cos\gamma + 2\sin\beta\sin\gamma + 2\cos\gamma\cos\alpha + 2\sin\gamma\sin\alpha + 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta = 0$

$\sin^2\alpha + \cos^2\alpha + \sin^2\beta + \cos^2\beta + \sin^2\gamma + \cos^2\gamma + 2\cos\beta\cos\gamma + 2\sin\beta\sin\gamma + 2\cos\gamma\cos\alpha + 2\sin\gamma\sin\alpha + 2\cos\alpha\cos\beta + 2\sin\alpha\sin\gamma = 0$

$(\sin\alpha + \sin\beta + \sin\gamma)^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2$

$\sin\alpha + \sin\beta + \sin\gamma = 0$ and $\cos\alpha + \cos\beta + \cos\gamma = 0$

55. (B) dividend = $6^{2x} - (34)^{2x+1} = 36^x - (34)^{2x+1}$

$= (35 + 1)^x - (35 - 1)^{2x+1}$

when it is dividend by 7,

Remainder = $1^x - (-1)^{2x+1} = 1 - (-1) = 2$

56. (B) Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$

Then, $\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4}$ ....

$\frac{2}{3}S = 1 + \left(\frac{2}{3} - \frac{1}{3}\right) + \left(\frac{6}{3^2} - \frac{2}{3^2}\right) + \left(\frac{10}{3^3} - \frac{6}{3^3}\right) + \left(\frac{14}{3^4} - \frac{10}{3^4}\right) + ...$

$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4}$ ....

$\frac{2}{3}S = \frac{\frac{4}{3}}{1 - \frac{1}{3}}$

$\frac{2}{3}S = 2$

$S = 3$

57. (D) $\cos(\alpha - \beta) = \frac{3}{5} \Rightarrow \tan(\alpha - \beta) = \frac{4}{3}$ and, $\sin(\alpha + \beta) = \frac{8}{17} \Rightarrow \tan(\alpha + \beta) = \frac{8}{15}$

Now, $\tan 2\alpha = \frac{\tan(\alpha - \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta).\tan(\alpha - \beta)} = \frac{\frac{4}{3} + \frac{8}{15}}{1 - \frac{4}{3} \times \frac{8}{15}} = \frac{84}{13}$

58. (C) In a 3 – D space, we know that,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 45° + \cos^2 120° + \cos^2\gamma = 1$

$\frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\cos^2\gamma = \frac{1}{4}$

Then, the angle made by AB with the positive z-axis = 60°

59. (D) $A = \sin^2 x + \cos^4 x$

$A = 1 - \cos^2 x + \cos^4 x$

$A = \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$

Here, $0 \le \cos^2 x \le 1$

So, $\frac{3}{4} \le A \le 1$

60. (A) 3sinA + 4 cosB = 6 .... (i)

and, 4sinB + 3cosA = 1 .... (ii)

Squaring and adding equation (i) and (ii), we get

$9\sin^2A + 16\cos^2B + 24\sin A\cos B + 16\sin^2B + 9\cos^2A + 24\cos A\sin B = 6^2 + 1^2$

$9(\sin^2A + \cos^2A) + 16(\sin^2B + \cos^2B) + 24(\sin A\cos B + \cos A\sin B) = 37$

$25 + 24\sin(A + B) = 37$

$24\sin(A + B) = 12$

$\sin(A + B) = \frac{1}{2}$

$\sin(\pi - C) = \frac{1}{2}$

$\sin C = \frac{1}{2} \Rightarrow C = 30°$

61. (B) Efficiency of A = 1

Efficiency of B = 1.5

Let B worked for x hours

Then,

$\frac{A \times 12}{\frac{5}{8}} = \frac{B \times x}{\frac{3}{8}}$

$3A \times 12 = 5B \times x$

$3 \times 1 \times 12 = 5 \times 1.5 \times x$

$x = \frac{36}{5 \times 1.5} = 4.8$ hours

62. (C)

5 men → $5 \times \frac{8}{5} = 8$ ⟍ 5

10 women → $6 \times \frac{100}{60} = 10$ ⟩ 40 ⟨ 4

8 children → $8 \times \frac{100}{40} = 20$ ⟋ 2

Now, efficiency of 5 men, 5 women and 4 children = $5 + \frac{4}{2} + \frac{2}{2} = 8$

Then, time taken to complete the work = $\frac{40}{8}$ = 5 hours

63. (C) Let distance between A and B = x km

According to the question,

Total time taken by car (t) = $\frac{x}{P_1} + \frac{x}{P_2} + \frac{x}{P_2}$

$t = \frac{P_2x + P_1x + P_1x}{P_1P_2}$

∴ Average speed at the car = $\frac{3x}{\frac{P_2x + 2P_1x}{P_1P_2}} = \frac{3P_1P_2}{P_2 + 2P_1}$

64. (B)

A	B	C
1200	1080	1080
1000	1000	950

Ratio of distance travelled by A, B and C = 1200 × 1000 : 1080 × 1000 : 1080 × 950
= 1000 : 900 : 855

When A travells 800 m, then distance travelled by C = $\frac{855}{1000} \times 800 = 684$ m

∴ The distance by which A beat C = 800 – 684 = 116 m

65. (B)

	Efficiency	time
Suresh	120	100
Mahesh	100	120

A.T.Q,
120 units = 30 days

1 unit = $\frac{1}{4}$ days

Then, time taken by Suresh to complete the work = 100 units = $100 \times \frac{1}{4}$ = 25 days

66. (A) Area of intersecting region = $(4\pi - 3\sqrt{3})\frac{r^2}{6} = \frac{1}{6}(4\pi - 3\sqrt{3})\text{cm}^2$

$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)\text{cm}^2$

67. (C) We know that,

Sum of the cubes of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

Sum of the cubes of first 15 natural numbers = $\left(\frac{15 \times (15+1)}{2}\right)^2 = \left(\frac{15 \times 16}{2}\right)^2 = 15^2 \times 8^2$

Then, average = $\frac{15^2 \times 8^2}{15}$ = 15 × 64 = 960

68. (C) $P\left[\frac{r}{100}\right]^2 = 81$

where, $r = \frac{15}{2}$

Then, $P\left[\frac{15}{200}\right]^2 = 81$

$P = \frac{81 \times 200 \times 200}{15 \times 15}$

P = ₹ 14400

∴ Principal amount = ₹14400

69. (D) $\sqrt{5x-14} + \sqrt{5x+14} = 7+\sqrt{21}$

Squaring both side, we get

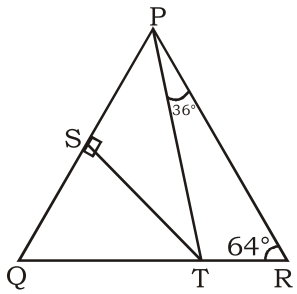
$5x - 14 + 5x + 14 + 2\sqrt{25x^2-14^2} = 49 + 21 + 2 \times 7\sqrt{21}$

On comparing we get,

$10x = 70$ and $2\sqrt{25x^2-196} = 14\sqrt{21}$

So, $x = 7$

70. (B)



We know that the sum of two interior angle is equal to the external angle

So, PTQ = 64° + 36° = 100°

Now, In ΔTQS,

$\angle T = \frac{100°}{2} = 50°$ and $\angle S = 90°$

Then, $\angle PQR = 180° - (50° + 90°) = 180° - 140° = 40°$

71. (B) Let the revolutions made during the journey be n.

Then,

$n \times 2\pi r = \frac{900000}{60} \times 55$

$n \times \frac{22}{7} \times 21 = \frac{900000 \times 55}{60}$

$n = \frac{900000 \times 55 \times 7}{60 \times 22 \times 21}$

n = 12500

72. (A) $\{\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3\}\frac{3}{4} = \frac{4}{3}\pi R^3$

$\frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3) \times \frac{3}{4} = \frac{4}{3}\pi R^3$

$(2^3 + 4^3 + 6^3) \times \frac{3}{4} = R^3$

R = 6

Then, diameter of the new ball = 2 × 6 = 12 cm

73. (D) $x \times \dfrac{2\tan^2 15°}{2-(1+\tan^2 15°)} = \cos 30° + \sin 60°$

$x \times \dfrac{2\tan^2 15°}{1-\tan^2 15°} = \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2}$

$x \times \tan 30° = \sqrt{3}$

$x \times \dfrac{1}{\sqrt{3}} = \sqrt{3}$

$x = 3$

74. (B) $25\% = \dfrac{1 \rightarrow \text{Profit}}{4 \rightarrow \text{CP}}$
Then, SP = 4 + 1 = 5
Now,
Number of pens bought for 4 rupees = 4 × 5 = 20
and, SP of 20 pens = ₹5
Then,

Number of pens sold for ₹5 = $\dfrac{20}{5}$ = 4

75. (A) In an equilateral triangle, the ratio of inradius and circumradius is 1 : 2.

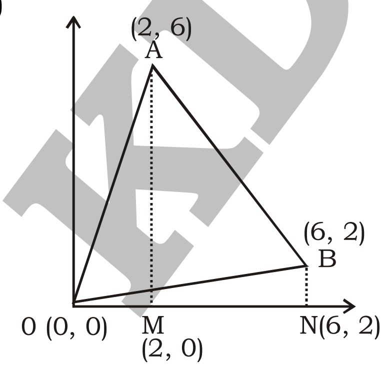
76. (C) $\dfrac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}} - \dfrac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = 112\sqrt{3}$

$\dfrac{\left(x+\sqrt{x^2-1}\right)^2-\left(x-\sqrt{x^2-1}\right)^2}{\left(x-\sqrt{x^2-1}\right)\left(x+\sqrt{x^2-1}\right)} = 112\sqrt{3}$

$\dfrac{4x\sqrt{x^2-1}}{x^2-\left(x^2-1\right)} = 112\sqrt{3}$

$x\sqrt{x^2-1} = 28\sqrt{3}$
Squaring both sides, we get
$x^2(x^2-1) = 28 \times 28 \times 3$
$x^4 - x^2 - 2352 = 0$
On solving, we get $x = 7$

77. (A)



Area of ΔOAB = ar(ΔOAM) + ar(AMNB) – ar(ΔOBN)

$= \dfrac{1}{2} \times 2 \times 6 + \dfrac{1}{2}(6+2) \times 4 - \dfrac{1}{2} \times 6 \times 2 = 16$ sq. units

78. (B) $p(x+y)^2 = 5$

$(x+y)^2 = \frac{5}{p}$

and, $q(x-y)^2 = 3$

$(x-y)^2 = \frac{3}{q}$

Now, $(x+y)^2 - (x-y)^2 = \frac{5}{p} - \frac{3}{q}$

$4xy = \frac{5q-3p}{pq}$

$4pqxy = 5q - 3p$

Then, $p^2(x+y)^2 + 4pqxy - q^2(x-y)^2 = 5p + (5q-3p) - 3q$

$= 5p + 5q - 3p - 3q = 2p + 2q = 2(p+q)$

79. (D) $x = \frac{5-2\sqrt{6}}{5+2\sqrt{6}}$

$x = \frac{(5-2\sqrt{6})(5-2\sqrt{6})}{(5+2\sqrt{6})(5-2\sqrt{6})}$

$x = 25 + 24 - 20\sqrt{6}$

similarly, $y = 49 + 20\sqrt{6}$

Then, $x + y = 98$

Squaring both sides, we get

$x^2 + y^2 + 2xy = 98^2$

$x^2 + y^2 = 98^2 - 2 = 9602$

80. (B) When medians of a triangle interest each other at right angle

Then, $AB^2 + AC^2 = 5BC^2$

$5BC^2 = 18^2 + 21^2$

$5BC^2 = 324 + 441$

$BC^2 = \frac{765}{5}$

$BC^2 = 153$

$BC = 3\sqrt{17}$ cm

81. (C) $x = 3^{\frac{1}{3}} + 3^{\frac{-1}{3}}$

Taking cube on both sides, we get

$x^3 = 3 + \frac{1}{3} + 3 \times 3^{\frac{1}{3}} \times 3^{\frac{-1}{3}}\left(3^{\frac{1}{3}} + 3^{\frac{-1}{3}}\right)$

$x^3 = \frac{10}{3} + 3x$

$3x^3 - 9x = 10$

$3x^3 - 9x - 8 = 2$

82. (A) Speed of runner A = $\frac{400}{80}$ = 5 m/s

and, speed of runner B = $\frac{200}{50}$ = 4 m/s

Now, Time taken by A to finish 1200 metre race = $\frac{1200}{5}$ = 240 sec.

Then, distance travelled by B in 240 sec. = 240 × 4 = 960 m

∴ Distance by which A beat B = 1200 – 960 = 240 m

83. (C) Let the numbers of markers of the institution be x.

Then,

$x \times 80 = 15 \times 600 + (x - 15)60$

$80x = 9000 + 60x - 900$

$80x - 60x = 9000 - 900$

$20x = 8100$

$x = 405$

∴ Total number of the workers of the institution = 405

84. (C) We know that,

$$P = \frac{x}{\left(1+\frac{r}{100}\right)} + \frac{x}{\left(1+\frac{r}{100}\right)^2} \ldots\ldots\ldots + \frac{x}{\left(1+\frac{r}{100}\right)^3}$$

where P = principal sum

x = amount of each instellment

r = rate of interest

and n = number of installments

Then, $25200 = x\left[\frac{20}{21}\right] + x\left[\frac{20}{21}\right]^2 + x\left[\frac{20}{21}\right]^3$

$25200 = x \times \frac{20}{21}\left[1+\frac{20}{21}+\frac{400}{441}\right]$

$x = \frac{25200 \times 21 \times 441}{20 \times 1261}$ = ₹ 9253.65

85. (C) Let $x = \sqrt{a+b} - \sqrt{a-b}$

where a = 3 and b = $\sqrt{-2+6\sqrt{2}}$

Squaring both sides, we get

$x^2 = (a + b) + (a - b) - 2\sqrt{a^2 - b^2}$

$x^2 = 2a - 2\sqrt{a^2 - b^2}$

On putting the respective values

$x^2 = 2 \times 3 - 2\sqrt{3^2 - \left(\sqrt{-2+6\sqrt{2}}\right)^2}$

$x^2 = 6 - 2\sqrt{9+2-6\sqrt{2}}$

$x^2 = 6 - 2\left(3-\sqrt{2}\right)$

$x^2 = 6 - 6 + 2\sqrt{2}$

$x^2 = 2\sqrt{2}$

$x = 2^{\frac{3}{4}}$

86. (A) Area of the regular octagon in a circle = $\frac{1}{2}r^2\sin 45° \times 8$

$= \frac{1}{2} \times 2 \times 2 \times \frac{1}{\sqrt{2}} \times 8 = 8\sqrt{2}$ sq. units

87. (D) $\sec\theta\,\text{cosec}\,\theta = \frac{1}{\cos\theta.\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta.\sin\theta} = \tan\theta + \cot\theta$

let $\tan\theta = x$ and $\cot\theta = \frac{1}{x}$

Then, $x^5 + \frac{1}{x^5} = 123$

$\left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) = 123$

$\left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - \left(x + \frac{1}{x}\right) = 123$

put $x + \frac{1}{x} = t$

$(t^3 - 3t)(t^2 - 2) - t = 123$

Using options; we get $t = 3$

$\therefore$ $\sec\theta\,\text{cosec}\,\theta = 3$

88. (B) S.P of the toy = $500 \times \frac{80}{100} \times \frac{95}{100}$ = ₹380

89. (C)

A	B
6000 × 8	8000 × 9
+10000 × 4	+3000 × 3
88000	81000

Ratio of profit of A and B = 88 : 81

Now, the amount which A takes as allowance = 150 × 12 = ₹ 1800

Then, Remaining profit = 10250 – 1800 = ₹ 8450

This profit is shared between A and B

So, share of B = $\frac{8450}{88+81} \times 81$ = ₹4050

90. (A) Total age reduced when an old student is replaced by a new student = 35 × 4 = 140 months

Then, age of the new student = 22 years – 140 months = 10 years 4 months

91. (D) Time taken to travel from A to C and back = 16 hours

Then, time taken to travel from A to B and back = 2 × 16 = 32 hours

Now, time taken to travel from B to A = $32 - 20\frac{1}{3} = 11\frac{2}{3}$ hours

92. (C) Percentage of candidates who passed in the examination = 82 + 75 – 78 = 79

Then, percentage of candidates who failed in the examination = 100 – 79 = 21

A.T.Q,

21% = 4200

Then, total number of candidates = 100% = $\frac{4200}{21} \times 100$ = 20000

93. (B)

	Milk	Water		
A	3	2	5	× 12×1
B	3	1	4	× 15×2
C	2	1	3	× 20×3

Now, New ratio -

	Milk		Water
A	36		24
B	90		30
C	120		60
	246	:	114
	41	:	19

Then, required ratio = 41 : 19

94. (D) 40% = $\frac{2}{5}$

Then, CP $\rightarrow$ 5
MP $\rightarrow$ 5 + 2 = 7

and 10% discount = $\frac{1}{10}$ $\rightarrow$discount, $\rightarrow$MP

So, MP $\rightarrow$ 10 and SP $\rightarrow$ 10 – 1 $\rightarrow$ 9

CP	MP	SP
5	7	7
10	10	9
50	70	63

Now, gain percent = $\frac{63-50}{50}\times100$ = 26%

95. (C) $\frac{2}{3}A = \frac{3}{5}B$

$\frac{A}{B} = \frac{9}{10}$

Now, (9 + 10) units = 2850
1 unit = 150
Then, profit of B = 10 units = 10 × 150 = ₹1500

96. (B) 72° = ₹1875

Then, monthly income of the family = 360° = $\frac{1875}{72}\times 360$ = ₹ 9375

97. (D) Percentage of savings = $\frac{108}{360}\times100$ = 30%

98. (B) Ratio of expenses on rent and food = 72 : 90 = 4 : 5

99. (A) Monthly income of the family (360°) = 15000

Now, average of expenses on rent, food and miscellaneous = $\frac{72+90+72}{3}$ = 78°

As, 360° = ₹ 15000

Then, 78° = $\frac{15000}{360}\times78$ = ₹ 3250

100. (C) Ratio of average of expenses on food, rent and miscellaneous items to the average of expenses on savings and clothing = $\frac{72+72+90}{3} : \frac{108+18}{2}$

$= \frac{234}{3} : \frac{126}{2}$ = 26 : 21

QUANTITATIVE ABILITY - 68 (ANSWER KEY)

1.	(B)	26.	(B)	51.	(D)	76.	(C)
2.	(D)	27.	(C)	52.	(C)	77.	(A)
3.	(A)	28.	(B)	53.	(B)	78.	(B)
4.	(B)	29.	(C)	54.	(A)	79.	(D)
5.	(C)	30.	(A)	55.	(B)	80.	(B)
6.	(B)	31.	(A)	56.	(B)	81.	(C)
7.	(C)	32.	(B)	57.	(D)	82.	(A)
8.	(B)	33.	(C)	58.	(C)	83.	(C)
9.	(D)	34.	(B)	59.	(D)	84.	(C)
10.	(C)	35.	(B)	60.	(A)	85.	(C)
11.	(A)	36.	(C)	61.	(B)	86.	(A)
12.	(B)	37.	(B)	62.	(C)	87.	(D)
13.	(D)	38.	(D)	63.	(C)	88.	(B)
14.	(C)	39.	(C)	64.	(B)	89.	(C)
15.	(B)	40.	(B)	65.	(B)	90.	(A)
16.	(C)	41.	(C)	66.	(A)	91.	(D)
17.	(A)	42.	(A)	67.	(C)	92.	(C)
18.	(C)	43.	(A)	68.	(C)	93.	(B)
19.	(B)	44.	(C)	69.	(D)	94.	(D)
20.	(D)	45.	(B)	70.	(B)	95.	(C)
21.	(C)	46.	(C)	71.	(B)	96.	(B)
22.	(B)	47.	(C)	72.	(A)	97.	(D)
23.	(D)	48.	(A)	73.	(D)	98.	(B)
24.	(C)	49.	(A)	74.	(B)	99.	(A)
25.	(C)	50.	(A)	75.	(A)	100.	(C)