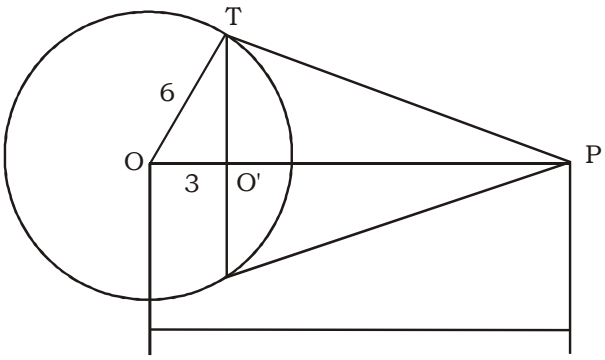
QUANTITATIVE ABILITY - 81 (SOLUTION)

1. (C) 2-D representation of the arrangement is following. O' is the center of the cone.
Let x be the distance of P from center of the sphere (O).
Height of the cone = x – 3
Base radius = OT



From triangle OO'T,

$OT = \sqrt{6^2 - 3^2} = \sqrt{27}$

From triangle OTP

$OP^2 = x^2 = 6^2 + PT^2$ .....(i)

From triangle TO'P

$PT^2 = (x - 3)^2 + 27$ .....(ii)

From equation (i) and (ii),

$(x - 3)^2 + 27 = x^2 - 36$

$x = 12$

So height of cone = (12 – 3) = 9 units

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(27)(9) = 81\pi$

2. (D) We have $\sin^3 10° + \sin^3 50° - \sin^3 70°$

Using the identity,

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$

Let us use this in the given equation:

$= \frac{1}{4}\left[(3\sin 10° - \sin 30°) + (3\sin 50° - \sin 150°)\right] - (3\sin 70° - \sin 210°)$

$= \frac{1}{4}\left[3(\sin 10° + \sin 50° + \sin 70°) - \frac{3}{2}\right]$

Now using,

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$= -\frac{1}{4}\left[3(\sin 10° - 2\cos 60°.\sin 10°) - \frac{3}{2}\right] = -\frac{3}{8}$

3. (D) $x^4 - 79x^2 + 1 = 0$

$$\frac{x^4}{x^2} + \frac{1}{x^2} = 79$$

$$x^2 + \frac{1}{x^2} = 79$$

$$x + \frac{1}{x} = \sqrt{x^2 + \frac{1}{x^2} + 2} = \sqrt{79 + 2} = 9$$

$$x^3 + x^{-3} = x^3 + \frac{1}{x^3}$$

$$= \left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right) = 9(79 - 1) = 702$$

4. (A) Amount of milk in container after taking 5 litres milk = 80 – 5 = 75 litres
Ratio of milk and water after adding 15 litres of milk = 75 : 15 = 5 : 1

Amount of milk in 18 litres mixture = $\frac{5}{6} \times 18$ = 15 litres

Amount of water in 18 litres mixture = $\frac{1}{6} \times 18$ = 3 litres

Remaining milk in the container = 75 – 15 = 60 litres
Remaining water in the container = 15 – 3 = 12 litres
Ratio of milk and water after adding 8 litres of water = 60 : (12 + 8)
= 60 : 20 = 3 : 1

Amount of milk in 28 litres mixture = $\frac{3}{4} \times 28$ = 21 litres

Remaining milk in container = 60 – 21 = 39 litres

5. (C) Let initially there are x inlets in the pool.
Now, according to the question x inlets fill the tank in 20 hours and (x – 12) inlets fill the tank in 32 hours.
ATQ,
20x = 32(x – 12)
20x = 32x – 384
12x = 384
x = 32
So, total number of inlets = 32
Now,
Flow rate of 1 inlet pipe is 2500 cubic metre per hour, so volume of the pool = flow rate of the pipe × number of pipes × time taken = 2500 × 32 × 20 = 16000000 cubic metre

6. (A) Let the length of each train be x m.

Relative speed = 60 – 45 = 15 kmph = $15 \times \frac{5}{18} = \frac{25}{6}$ m/s

Now, Time = $\frac{\text{Distance}}{\text{Speed}}$

$$24 = \frac{x + x}{\frac{25}{6}}$$

$$24 = 2x \times \frac{6}{25}$$

$$x = \frac{25 \times 24}{6 \times 2} = 50 \text{ m}$$

Short trick :

When two train of same length running opposite direction, then length of each train

$$= \frac{\text{Relative speed} \times \text{time to cross each other}}{2} = \frac{15 \times \frac{5}{18} \times 24}{2} = 50 \text{ m}$$

7. (A) Let the investment made by Sapna be ₹ x.
Then, investment made by Neha = (81600 – x)
ATQ,

$$(81600 - x)\left(1 + \frac{4}{100}\right)^2 = x\left(1 + \frac{4}{100}\right)^3$$

$$81600 - x = 1.04x$$

$$x = \frac{81600}{2.04} = ₹\ 40{,}000$$

8. (B) Using Alligation Method,

Rice I	**Rice II**
5.75	4.50

5.50

5.50 – 4.50 = 1.00 i.e., 4 : 1

5.75 – 5.50 = 0.25

Hence, the required quantity of Rice I = $\frac{75}{1} \times 4 = 300$ kg

9. (B) Let the share of A be ₹ x.
Then, the share of B is ₹ (30600 – x)
ATQ,

$$x \times \left(1 + \frac{4}{100}\right)^3 = (30600 - x)\left(1 + \frac{4}{100}\right)^2$$

$$x \times \frac{104}{100} = 30600 - x$$

$$\frac{204}{100}x = 30600$$

$$x = \frac{30600 \times 100}{204} = ₹\ 15{,}000$$

10. (D) Let the length of each train be x metre.

Relative speed = 46 – 36 = 10 km/h = $10 \times \frac{5}{18}$ m/s = $\frac{25}{9}$ m/s

ATQ,

$$\frac{x + x}{\frac{25}{9}} = 72$$

$$\frac{2x \times 9}{25} = 72$$

x = 100 m

11. (B) C.P. of land = ₹ 96000

Loss at $\frac{2}{5}$ th land = 6%

$$= \frac{2}{5} \times \frac{6}{100} \times 96{,}000 = ₹\ 2304$$

Overall profit = 10%

$$= \frac{10}{100} \times 96{,}000 = ₹\ 9600$$

Profit of $\frac{3}{5}$ th land = ₹ 9600 + ₹ 2304 = ₹ 11904

$$\text{Profit percent} = \frac{11904}{\frac{3}{5} \times 96{,}000} \times 100 = 20\frac{2}{3}\%$$

12. (D) Sunil 4 — 3
Dinesh 6 — 2
(12)

Work done by Ramesh in one day = $3 \times \frac{3}{2} = \frac{9}{2}$

$$\text{Time taken by all of them together} = \frac{12}{3 + 2 + 4\frac{1}{2}} = \frac{24}{19} = 1\frac{5}{19} \text{ days}$$

13. (C) $x^3 + y^3 + z^3 - 3xyz$

$$= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

$$= \frac{1}{2}(333 + 333 + 334)[(0)^2 + (-1)^2 + (-1)^2] = 1000$$

14. (A) $\frac{x-a^2}{b+c}+\frac{x-b^2}{a+c}+\frac{x-c^2}{a+b}=4(a+b+c)$

$\frac{x-a^2}{b+c}-a+\frac{x-b^2}{a+c}-b+\frac{x-c^2}{a+b}-c=3(a+b+c)$

$\frac{x-a^2-a(b+c)}{b+c}+\frac{x-b^2-b(a+c)}{a+c}+\frac{x-c^2-c(a+c)}{a+b}=3(a+b+c)$

$\frac{x-a^2(a+b+c)}{b+c}+\frac{x-b^2(a+b+c)}{a+c}+\frac{x-c^2(a+b+c)}{a+b}=3(a+b+c)$

$\frac{x}{b+c}-\frac{(a+b+c)a}{b+c}+\frac{x}{a+c}-\frac{(a+b+c)b}{a+c}+\frac{x}{a+c}-\frac{(a+b+c)}{a+b}=3(a+b+c)$

$\frac{x}{(b+c)(a+b+c)}-\frac{a}{b+c}+\frac{x}{(a+b+c)(a+c)}-\frac{b}{b+c}+\frac{x}{(a+b+c)(a+c)}-\frac{c}{a+c}=3$

$\frac{x}{(a+b+c)}\left[\frac{1}{b+c}+\frac{1}{a+c}+\frac{1}{a+b}\right]-\frac{a}{b+c}\frac{b}{a+c}-\frac{c}{a+b}=3$

$\frac{x}{a+b+c}\left[\frac{1}{b+c}+\frac{1}{a+c}+\frac{1}{a-b}\right]-\frac{a}{b+c}-1-\frac{b}{a+c}-\frac{c}{a+b}-1=0$

$\frac{x}{a+b+c}\left[\frac{1}{b+c}+\frac{1}{a+c}+\frac{1}{a+b}\right]-\frac{(a+b+c)}{b+c}-\frac{(a+b+c)}{a+c}-\frac{a+b+c}{a+b}=0$

$\left[\frac{1}{b+c}+\frac{1}{a+c}+\frac{1}{a+b}\right]\left[\frac{x}{a+b+c}-(a+b+c)\right]=0$

So,

$\frac{x}{a+b+c}-(a+b+c)=0$

$x=(a+b+c)^2$

15. (D) $(x-a)(x-b)=1$

$(x-a)(x-a-5)=1$ $[a-b+5=0]$

$x-a-5=\frac{1}{x-a}$

$(x-a)-\frac{1}{(x-a)}=5$

Cubing both side,

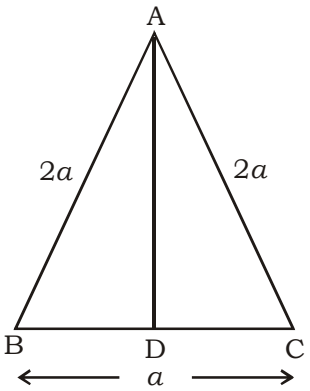
$(x-a)^3-\frac{1}{(x-a)^3}-(x-a)\left(\frac{1}{x-a}\right)=125$

$\left[\left((x-a)-\frac{1}{x-a}\right)\right]=125$

$(x-a)^3-\frac{1}{(x-a)^3}=125+3(5)$

$(x-a)^3-\frac{1}{(x-a)^3}=140$

16. (B)

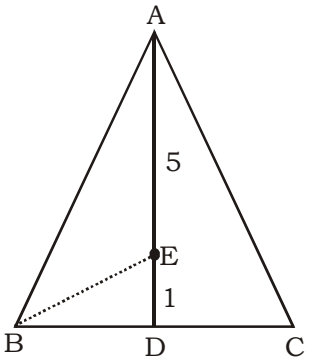


$BD = \frac{1}{2}BC = \frac{1}{2}a$ unit

$AD = \sqrt{(AB)^2 - (BD)^2} = \sqrt{(2a)^2 - \left(\frac{1}{2}a\right)^2}$

$= \sqrt{4a^2 - \frac{1}{4}a^2} = \frac{\sqrt{15}}{2}a$ unit

17. (C)



AE : ED = 5 : 1

Let AE = $5a$ unit

Then,

ED = a unit

$\angle BAD = 30°$ [∴ given]

$BD = AD \times \tan 30°$

$= 6a \times \frac{1}{\sqrt{3}} = 2\sqrt{3}$ a unit

$\tan(\angle ACB) = 6 \tan(\angle DBE)$

In ΔACD,

$\tan(\angle ACB) = \frac{AC}{CD}$ $\tan(\angle DBE) = \frac{DE}{BD}$

$\frac{AD}{CD} = 6 \times \frac{DE}{BD}$

$CD = \frac{6a \times 2\sqrt{3}a}{6 \times a} = 2\sqrt{3}a$ unit

$BC = BD + DC = 4\sqrt{3}\,a$ unit

$AB = \frac{BD}{\sin 30^\circ} = \frac{2\sqrt{3}\,a}{\frac{1}{2}} = 4\sqrt{3}\,a$ unit

$AC = \sqrt{AD^2 + CD^2} = \sqrt{(6a)^2 + (2\sqrt{3}a)^2} = 4\sqrt{3}\,a$ unit

$AB = BC = AC = 4\sqrt{3}\,a$ unit [Property of equilateral triangle]

So, $\angle ACB = 60^\circ$

18. (B) Let Auto rickshaw charge for the distance covered = ₹ x / km and fixed charge = ₹ y
According to given condition,
$10x + y = 85$ ..... (i)
$15x + y = 120$ ..... (ii)
Solving these equations (i) and (ii), we get
x = ₹ 7, y = ₹ 15
Hence, fare for journey of 25 km = ₹ $(25x + y)$
= ₹(25 × 7 + 15) = ₹ 190

19. (B) Let $p(x) = ax^3 + bx^2 + x - 6$ be the given polynomial.
Now,
$(x + 2)$ is a factor of $p(x)$.
$p(-2) = 0$ [$\because x + 2 = 0 \Rightarrow x = -2$]
$a(-2)^3 + b(-2)^2 + (-2) - 6 = 0$
$-8a + 4b - 2 - 6 = 0$
$-8a + 4b = 8$
$-2a + b = 2$ ... (i)
It is given that $p(x)$ leaves the remainder 4 when it is divisible by $(x - 2)$.
$\therefore$ $p(2) = 4$
$a(2)^3 + b(2)^2 + 2 - 6 = 4$
$8a + 4b - 4 = 4$
$8a + 4b = 8$
$2a + b = 2$ ... (ii)
Adding (i) and (ii), we get
$2b = 4$
$b = 2$
Putting b = 2 in (i), we get
$-2a + 2 = 2$
$-2a = 0$
$a = 0$
Hence, $a = 0$, $b = 2$

20. (C) In AP, there is a, b, c.

$\therefore$ $b - a = c - b$

In GP, there is x, y, z.

$y^2 = xz$

$x^{b-c}\, y^{c-a}\, z^{a-b} = x^{a-b}\, z^{a-b} \times y^{c-a}$

$= (xz)^{a-b} \times y^{c-a} = y^{2(a-b)} \times y^{c-a}$

$= y^{2a-2b+c-a} = y^{a+c-2b}$

$= y^{2b-2b} = y^0 = 1$

21. (A) Let number of postcards and inland letters be x and y respectively.

ATQ,

$0.30x + 1.5y = 66$ .....(i)

and $x + y = 60$ .....(ii)

On solving Eqs. (i) and (ii), we get

$x = 20$, $y = 40$

$\therefore$ Required cost price = $0.30 \times 40 + 1.5 \times 20$

$= 12 + 30 = ₹\ 42$

22. (B) Let length and breadth of the rectangular park be $3x$ and $2x$ respectively.

Then,

Distance = speed × time

ATQ,

$$10x = 12 \times \frac{5}{8} \times 8 \times 60$$

$x = 160$

$\therefore$ Area of rectangular park = $3x \times 2x = 6x^2$

$= 6 \times (160)^2 = 153600$ sq. m.

23. (A) The minimum distance each person should walk so that they can cover the distance in complete steps = LCM of (80 cm, 85 cm, 90 cm)

5	80, 85, 90
2	16, 17, 18
	8, 17, 9

Hence, the required distance = $5 \times 2 \times 8 \times 17 \times 9$

= 12240 cm = 122 m 40 cm

24. (A) Required difference = $[18 + (-4) + 28.3 + 15 + (-3.1) + (-18.8)] \times 100$

$= 35.4 \times 100 = 3540$

25. (D) Required ratio = $\dfrac{(65+71.6)\times 100}{(42+76)\times 100} = \dfrac{13660}{11800} = 683 : 590$

26. (C) Required total average = $\left[\dfrac{(65+41.2+72.4+63.5+83)\times 100}{5}\right] + \left[\dfrac{(51+72.8+83.5+21.8+66)\times 100}{5}\right]$

$= 6502 + 5902 = 12404$

27. (D) Required average = $\dfrac{(72.4+61+71.6+83.5+61.2+73.2)\times 100}{6} = 7048.33 \approx 7048$

28. (D) Required % = $\dfrac{32970}{34850} \times 100 \approx 95\%$

29. (A) $(x^2 + 2x + 1) + y^2 = 0$

$(x + 1)^2 + y^2 = 0$

$x + 1 = 0$

$x = -1$

$y = 0$ [By equating to zero]

$x^{31} + y^{35} = (-1)^{31} + (0)^{35}$

$= -1 + 0 = -1$

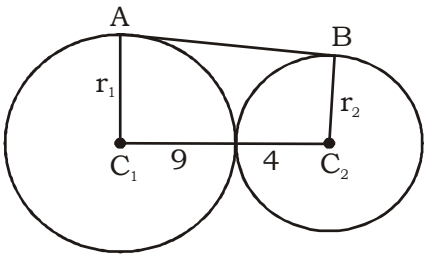
30. (A) Let, $\alpha = 2x$, $\beta = x$

$\alpha + \beta = 3x = 90$

$\therefore \quad \alpha = 60°, \beta = 30°$

$$\frac{\sin\alpha}{\sin\beta} = \frac{\sin 60°}{\sin 30°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} : 1$$

31. (C)



$r_1 = 9$ cm

$r_2 = 4$ cm

$C_1C_2 = 13$ cm

$\therefore$ Length $= \sqrt{(C_1C_2)^2 - (r_1 - r_2)^2}$

$= \sqrt{(13)^2 - (9-4)^2} = \sqrt{169 - 25}$

$= \sqrt{144} = 12$ cm

32. (B) Let BOC be the circular entrenchment which is surrounded by a ditch all around AD width of ditch = CD = W = 8 m

So, the ditch is a circular ring, with inner diameter = d = 54 m

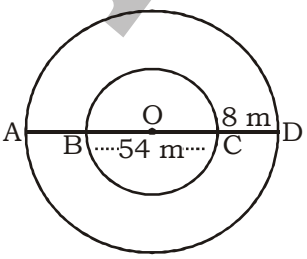
Thickness = W = 8 m

Cross section CD is a trapezium in shape.

Using the formula for ring,

Volume V = Al

Where A = area of cross-section (CD)



Since the cross-section CD is a trapezium having parallel sides

a = 8 m, b = 6 m

And perpendicular distance between the parallel sides = h = 5 m

$$A = \frac{1}{2}(a + b)h = \frac{1}{2}(8 + 6) \times 5 = 35 \text{ m}^2$$

l = length = $\pi(d + w)$

$$= \frac{22}{7}(54 + 8) = \frac{22 \times 62}{7} \text{ m}$$

Now, $V = Al = 35 \times \frac{22 \times 62}{7} \text{ m}^3 = 6820 \text{ m}^3$.

Hence, the volume of the excavation in digging the ditch is 6820 m^3.

33. (C) Using the formula for prism,

Whole surface area = $S = 2A + P_n \times h$

Where A = area of base

P_b = perimeter of base = 21 + 20 + 13 = 54 m

h = height = 30 m

A = area of triangle = $\sqrt{S(S-a)(S-b)(S-c)}$ (a = 21, b = 2, c = 13, S = 27)

$= \sqrt{27 \times 6 \times 7 \times 14}$ m^2 = 126 m^2

Now,

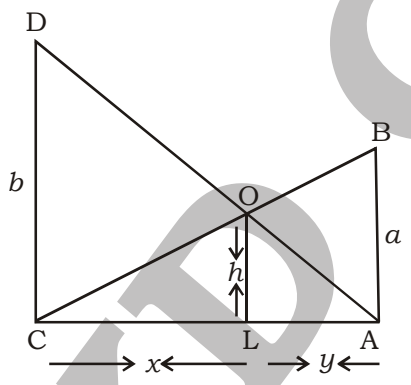
$S = 2A + P_b h$

$= 2 \times 126 + 54 \times 30 \text{ m}^2$

$= 252 + 1620 = 1872 \text{ m}^2$

Hence, the area of the whole surface of the given triangular prism is 1872 m^2.

34. (D)



Let, CL = x and LA = y

Then,

$x + y = p$

In ΔABC and ΔLOC,

$\angle CAB = \angle CLO$ [Each equal to 90°]

$\angle C = \angle C$ [common]

$\therefore$ $\Delta CAB \sim \Delta CLO$ [By AA-criterion of similarity]

$$\frac{CA}{CL} = \frac{AB}{LO}$$

$$\frac{p}{x} = \frac{a}{h}$$

$$x = \frac{ph}{a}$$

In ΔALO and ΔACD, we have [Each equal to 90°]

$\angle ALO = \angle ACD$ [common]

$\angle A = \angle A$ [By AA criterion of similarly]

$\therefore$ $\Delta ALO \sim \Delta ACD$

$$\frac{AL}{AC} = \frac{OL}{DC}$$

$$\frac{y}{p} = \frac{h}{b}$$

$$y = \frac{ph}{b}$$

From (i) and (ii), we have

$$x + y = \frac{ph}{a} + \frac{ph}{b}$$

$$p = ph\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$1 = h\left(\frac{a+b}{ab}\right)$$

$$h = \frac{ab}{a+b} \text{ metres}$$

Hence, the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is $\frac{ab}{a+b}$ metres.

35. (D) The given equation is:

$$\frac{x+a}{x-a} - \frac{x-b}{x-b} = \frac{2(a+b)}{x}$$

$$\frac{x+a}{x-a} - 1 - \frac{x-b}{x+b} + 1 = \frac{2(a+b)}{x}$$

$$\left(\frac{x+a}{x-a} - 1\right) - \left(\frac{x-b}{x+b} - 1\right) = \frac{2(a+b)}{x}$$

$$\frac{a}{x-a} + \frac{b}{x+b} = \frac{a+b}{x}$$

$$\frac{a}{x-a} + \frac{b}{x+b} = \frac{a}{x} + \frac{b}{x}$$

$$\frac{a}{x-a} - \frac{a}{x} = \frac{b}{x} - \frac{b}{x+b}$$ [After transposing]

$$\frac{ax - ax + a^2}{x(x-a)} = \frac{bx + b^2 - bx}{x(x+b)}$$

$$\frac{a^2}{x-a} = \frac{b^2}{x+b}$$

$a^2x + a^2b = b^2x - ab^2$ [After cross-multiplication]

$x(b^2 - a^2) = ab(a + b)$

$$x = \frac{ab}{b-a}$$

36. (C) $\sin(60° - \theta) = \cos(\psi - 30°)$

$\sin(60° - \theta) = \sin[90° - (\psi - 30°)]$

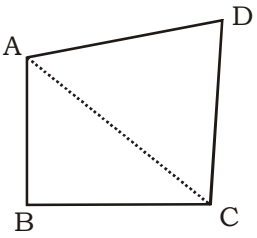
$\sin(60° - \theta) = \sin[90° - \psi + 30°]$

$60° - \theta = 120° - \psi$

$\psi - \theta = 60°$

$\tan(\psi - \theta) = \tan 60° = \sqrt{3}$

37. (C)



Since, $\angle B = 90°$,

In ΔABC,

$AC^2 = AB^2 + BC^2$

But it is known that

$AD^2 = AB^2 + BC^2 + CD^2$

$AD^2 = AC^2 + CD^2$

$\angle ACD = 90°$

In ΔACD,

Hence, $\angle ACD = 90°$.

38. (A) Marks obtained by Sushant = 1080

Marks obtained by Mohit = $1.2 \times 1080 = 1296$

Marks obtained by Rajesh = $\frac{1296}{0.9} = 1440$

So, percentage of marks obtained by Rajesh = $\frac{1440}{2000} \times 100 = 72\%$

39. (C) 27 cogs turns 80 times in $\frac{3}{4} \times 60$ min = 45s

27 cogs – 45s = 80 times

16 cogs – 8s = $\frac{27 \times 80 \times 8}{16 \times 45} = 24$ times

40. (C) Number of students who play at least one game = 100% – 52% = 48%
Let x% be the number of students who plays both the game.
ATQ,
40% + 34% – x% = 48%
x% = 26%
ATQ,
26% = 234
So, total number of students = 100% = 900

41. (B) Let the original cost price be ₹ 100.
Then,
Profit = ₹ 320 and SP = ₹ 420
New CP = ₹ 125
New profit = ₹ 295

Required percentage = $\frac{295}{420} \times 100 = 70\%$

42. (A) Required difference = $8.6 \times \frac{22}{100} - 5.4 \times \frac{15}{100} = 1.892 - 0.81$
= 1.082 lakh = 108200

43. (D) Vacancies in state R in the year 2016 = $5.4 \times \frac{10}{100}$ = 0.54 lakh

Vacancies in state R in the year 2017 = $8.6 \times \frac{8}{100}$ = 0.688

Required average = $\frac{0.54 + 0.688}{2} = \frac{1.228}{2}$ lakh = 61400

44. (C) Required sum = $5.4 \times \frac{8}{100} + 8.6 \times \frac{18}{100}$ = 0.432 + 1.548 = 1.98 lakh

45. (D) Total number of vacancies in 2017 = $\frac{48000 \times 100}{6}$ = 800000

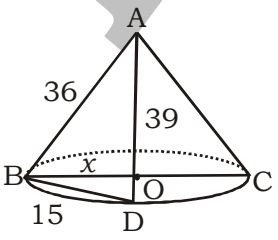
Vacancies in city B = 20% of 800000 = 160000 = 1.60 lakh

46. (C) Vacancies in state R in the year 2016 = $5.4 \times \frac{10}{100}$ = 0.54 lakh

Vacancies in state R in the year 2017 = $8.6 \times \frac{8}{100}$ = 0.688 lakh

Percentage rise = $\left(\frac{0.688 - 0.54}{0.54}\right) \times 100 = 27.407\% \approx 27\%$

47. (B)



$l = \sqrt{36^2 + 15^2}$ = 39 cm

If r = radius of one generated cone, then by property of similar triangles.

$\frac{r}{15} = \frac{36}{39}$

r = 13.85 cm

l_1 = 36 cm and l_2 = 15 cm

Volume of double = cone (above OB + below OB) = $\frac{1}{3}A(h_1 + h_2)$

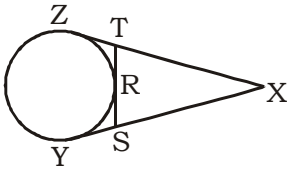
$= \frac{1}{3} \times \pi \times (13.85)^2 \times 39 = 7837 \text{ cm}^3$

48. (C) Total investment by A = 16000 × 3 + 11000 × 9 = ₹ 147000
Total investment By B = 12000 × 3 + 17000 × 9 = ₹ 189000
Total investment by C = 21000 × 6 = ₹ 126000
Ratio of investments = 147000 : 189000 : 126000 = 7 : 9 : 6

Profit of B exceeds that of C by $\frac{9-6}{7+9+6} \times 26400$

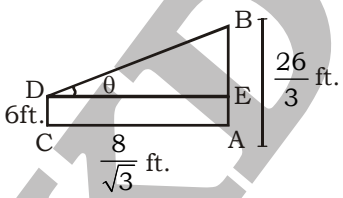
$= \frac{3}{22} \times 26400$ = ₹ 3600

49. (D)



XY = 15 cm
TX = 9 cm
XY = XZ [∵ Tangents from external point are equal]
XZ = 15 cm
Now, ZT = XZ – TX = 15 – 9 = 6 cm
RT = ZT = 6 cm

50. (C)



In ΔDBE,

BE = $\frac{26}{3} - 6 = \frac{8}{3}$

$\tan\theta = \frac{BE}{DE} = \frac{\frac{8}{3}}{\frac{8}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$

$\theta = 30°$

51. (C) Let the speed of boat in still water is x kmph and speed of stream is y kmph.

$x + y = 14$ ...(i)

and $x - y = 16$ ...(ii)

From (i) and (ii), we get

$x = 10$ kmph and $y = 4$ kmph

Required time = $\frac{40}{10}$ = 4 hours

Short trick :

Required time = $\frac{\text{Distance}}{\text{speed in still water}} = \frac{40}{\frac{\text{speed (down)+speed (up)}}{2}}$

$= \frac{40}{\frac{14+6}{2}} = \frac{40}{10}$ = 4 hrs.

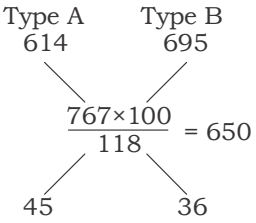
52. (A) Let their initial investments be ₹ x, ₹ $3x$ and ₹ $5x$ respectively.

Then, A : B : C = $(x \times 4 + 2x \times 8) : (3x \times 4 + \frac{3x}{2} \times 8) : (5x \times 4 + \frac{5x}{2} \times 8)$

$= (4x + 16x) : (12x + 12x) : (20x + 20x)$

$= 20x : 24x : 40x = 5 : 6 : 10$

53. (D)



Ratio of A : B = 5 : 4

So, type B sugar will be (7 × 4) = 28 kg

54. (C) Let speed of motorboat in still water be x km/h and speed of stream be y km/h.
Now, according to the question,

$\frac{25}{x-y} + \frac{39}{x+y} = 8$ ..... (i)

$\frac{35}{x-y} + \frac{52}{x+y} = 11$ ..... (ii)

By equation (i) × 4 – (ii) × 3,

We have,

$\frac{100}{x-y} - \frac{105}{x-y} = 32 - 33$

$\frac{-5}{x-y} = 1$

$x - y = 5$ ..... (iii)

From equation (i),

$$\frac{25}{5} + \frac{39}{x+y} = 8$$

$$\frac{39}{x+y} = 8 - 5 = 3$$

$x + y = 13$ ..... (iv)

By equation (iv) – (iii),

$x + y - x + y = 13 - 5 = 8$

$2y = 8$

$$y = \frac{8}{2} = 4 \text{ km/h}$$

55. (A) The quadratic equation is x^2 –(sum of roots)x + products of roots = 0

$x^2 - (2 + \sqrt{5} + 2 - \sqrt{5})\, x + (2 + \sqrt{5})(2 - \sqrt{5}) = 0$

$x^2 - 4x - 1 = 0$

56. (A) $\frac{x+1}{x-1} + \frac{x-1}{x+1} = \frac{10}{3}$

$$\frac{(x+1)^2 + (x-1)^2}{x^2 - 1} = \frac{10}{3}$$

$$\frac{2x^2+2}{x^2-1} = \frac{10}{3}$$

$(x^2+1)3 = 5\,(x^2-1)$

$3x^2+3 = 5x^2 - 5$

$8 = 2x^2$

$x^2 = 4$

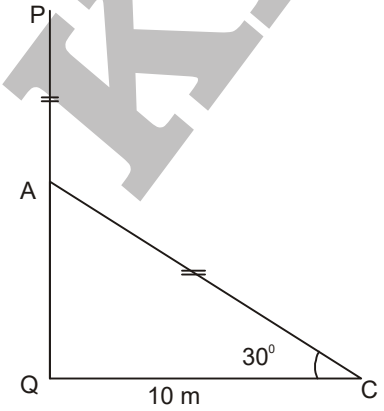
$x = \pm 2$

57. (C)

$$\begin{array}{r|l} & -2x+5 \\ \hline 2x+1 & -4x^2+8x+5 \\ & -4x^2-2x \\ & \hline & 10x+5 \\ & 10x+5 \\ & \hline & 0 \end{array}$$

Other factor = $5 - 2x$

58. (D) Let PQ be the tree.



$\frac{QC}{AC} = \cos 30^0$

$\frac{10}{AC} = \frac{\sqrt{3}}{2}$

$AC = \frac{20}{\sqrt{3}} m$

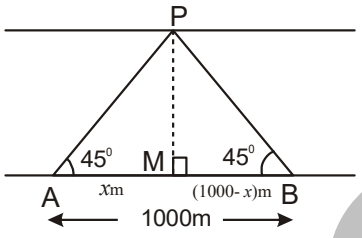
$\frac{AQ}{QC} = \tan 30^0$

$\frac{AQ}{10} = \frac{1}{\sqrt{3}}$

$AQ = \frac{10}{\sqrt{3}} m$

Height of the tree = $\frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = 10\sqrt{3}$ m

59. (D)



In ΔPMA

$\tan 45^0 = \frac{PM}{AM}$

$1 = \frac{PM}{AM}$

PM = xm

In ΔPMB

$\tan 45^0 = \frac{PM}{MB}$

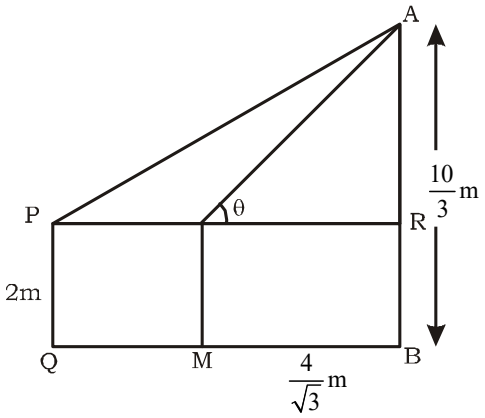
$1 = \frac{x}{1000 - x}$

$x = 1000 - x$

$x = 500$ m

Width of the river = PM = xm = 500m = $\frac{1}{2}$ km

60. (C)



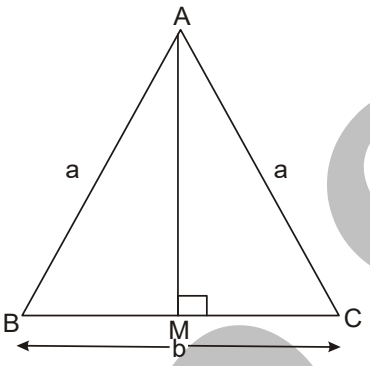
$AR = \frac{10}{3} - 2 = \frac{4}{3}$ m

In ΔAQR

$$\frac{AR}{RQ} = \frac{\frac{4}{3}}{\frac{4}{\sqrt{3}}} = \frac{1}{\sqrt{3}} = \tan\theta$$

$\theta = 30^0$

61. (C)



ΔABC is isosceles where
AB = AC = a and BC = b
Draw AM $\perp$ BC

$BM = MC = \frac{1}{2}BC$

$$AM = \sqrt{AB^2 - BM^2} = \sqrt{a^2 - \left(\frac{b}{2}\right)^2}$$

$$\text{Area of D ABC} = \frac{1}{2} \times b \times \sqrt{a^2 - \frac{b^2}{4}} = \frac{b}{2} \times \frac{\sqrt{4a^2 - b^2}}{2}$$

$$= \frac{b}{4}\sqrt{4a^2 - b^2}\ \text{sq.unit}$$

62. (C) If the C.P. of wrist watch be ₹ x, then C.P. of wall clock = ₹ $(390 - x)$
ATQ,

$$\frac{x \times 10}{100} + \frac{(390 - x) \times 15}{100} = 51.50$$

$10x + 5850 - 15x = 5150$
$5x = 5850 - 5150 = 700$

$$x = \frac{700}{5} = ₹\ 140$$

C.P. of wall clock = 390 – 140 = ₹ 250
∴ Required difference = 250 – 140 = ₹ 110

63. (D) Let the C.P = x
So, S.P. in 1st case = $1.05x$
Now, C.P. in 2nd case = $0.95x$
And S.P. in 2nd case = $1.05x - 2$
Now, A.T.Q. $0.95x \times 1.1 = 1.05x - 2$
$1.045x = 1.05x - 2$
$1.05x - 1.045x = 2$
$0.005x = 2$

$$x = \frac{2}{0.005} = ₹400$$

64. (C) Discount on ₹ 36000 = $\frac{3600 \times 7}{100}$ = ₹ 2520

Discount on first ₹ 20,000 = $\frac{20000 \times 8}{100}$ = ₹ 1600

Discount on next ₹ 10,000 = $\frac{10,000 \times 5}{100}$ = ₹ 500

Discount on remaining ₹ 6,000 = 2520 – (1600 + 500) = ₹ 420

∴ Required percent = $\frac{420 \times 100}{6000}$ = 7%

65. (D) In the race between Sonu and Monu.
Distance travelled by Sonu and Monu in same time = 600 mtr. and (600 – 60) mtr
= 600 mtr. and 540 mtr.
In the same time,
Ratio of distance travelled by Sonu & Monu = 10 : 9
Similarly, In the same time,
Ratio of distance travelled by Monu & Bablu = 500 : (500 – 25)
= 500 : 475 = 20 : 19
So, In the same time,
Ratio of distance travelled by Sonu, Monu & Bablu = 10 × 20 : 9 × 20 : 9 × 19
= 200 : 180 : 171
When Sonu travels 200 m, Bablu will travel 171 m
So, When Sonu travels 400 m, Bablu will travel 342 m
In 400 m race between Sonu & Bablu
Required Number of metres by which Sonu will win the race = 400 m – 342 m = 58 m

66. (A) Let x mtr = length of the faster train

So, 36 seconds = $\frac{x+10}{(40-20)\text{kmph}}$

x = 36 second × 20 × $\frac{5}{18}$ m/sec = 200 mtr.

67. (C) Increase in water level = $\frac{\text{Volume of sphere}}{\text{Area of base of cylinder}}$

= $\frac{\frac{4}{3}\pi r^2}{\pi r^2} = \frac{4}{3}r = \frac{4}{3}\times 3.5 = \frac{14}{3}$ cm.

∴ Required water level = $7 - \frac{14}{3} = \frac{7}{3}$ cm

68. (A) Curved surface of cylinder = $2\pi rh$

Radius = $\frac{1}{3}$r , height = 6h

Curved surface = $2\pi \times \frac{1}{3}$r ×6h = $(2\pi rh)\times 2$

∴ Increase will be twice.

69. (D) Let his deposit = ₹ 100

Interest for first 2 years = ₹ 6

Interest for first 3 years = ₹ 24

Interest for the last year = ₹ 10

Total interest = ₹ 40

When interest is ₹ 40, deposited amount is ₹100.

When interest is ₹1520, deposited amount = $\frac{100}{40}\times$ 1520 = ₹ 3800

Trick :

Principal = $\frac{Interest\times 100}{t_1r_1+t_1r_2+t_3r_3+....} = \frac{1520\times 100}{2\times 3+3\times 8+1\times 10}$

= $\frac{1520\times 100}{40}$ = ₹ 3800

70. (A) $\tan(x+y)\cdot\tan(x-y)=1$

$\tan(x+y) = \cot(x-y)$

$\tan(x+y) = \tan[90^\circ-(x-y)]$

$x+y = 90^\circ - x + y$

$2x = 90^\circ$

$x = 45^\circ$

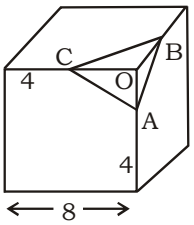
$\frac{2x}{3} = \frac{90}{3} = 30^\circ$

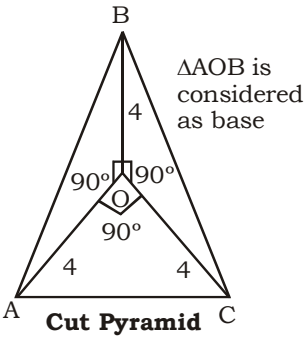
∴ $\tan\left(\frac{2x}{3}\right) = \tan 30^\circ = \frac{1}{\sqrt{3}}$

71. (B) The edge of the given cube = 8 m

Since the plane bisects the conterminous edges.

So, OA = OB = OC = 4 m





Cut Pyramid

The pyramid cut by the above plane has been shown in the second figure.

The three sides of the pyramid are right angled isosceles triangle because

$\angle BOA = \angle BOC = \angle AOC = 90°$

Taking one of the above triangles as base of the pyramid whose equal sides measure 4 m, the height = h = OB = 4 m

Using the formula,

Where,

A = area of $\Delta AOC = \frac{1}{2} \times 4 \times 4$

h = OB = 4 m

$V = \frac{1}{3} \times \frac{1}{2} \times 4^2 \times 4 \text{ m}^3 = 10.67 \text{ m}^3$

Hence, the volume of the cut pyramid is 10.67 m^3.

72. (C) $x^2 - 3x + 2 = (x - 2)(x - 1)$ are factors of $f(x) = 0$.

$f(2) = 0$

$2^5 - 5 \times 2^4 + A \times 2^3 + B \times 2^2 + 4 \times 2 - 40 = 0$

$8A + 4B = 80$

$2A + B = 20$ ..... (i)

Again, $f(1) = 0$

On further solving, we get

$A + B = 40$ ..... (ii)

From (i) and (ii),

$A = -20, B = 60$

73. (D) $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$

$\frac{a}{1-a} + 1 + \frac{b}{1-b} + 1 + \frac{c}{1-c} + 1 = 1 + 3$

$\frac{a+1-a}{1-a} + \frac{b+1-b}{1-b} + \frac{c+1-c}{1-c} = 4$

$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$

74. (A) Part of the tank filled in 2 min = $\left(\frac{2}{3}+\frac{2}{6}-\frac{2}{4}\right)=\frac{1}{2}$

P and Q can fill $\frac{1}{2}$ part of tank in = $\frac{1}{2}\times\left(\frac{3\times6}{3+6}\right)$ = 1 min

So, tank will be full in 3 min

75. (A) Given,

HCF = 4, LCM = 27720

n = 5

According to the formula,

Required product = $(HCF)^{n-1} \times LCM = (4)^{5-1} \times 27720$

$= (4)^4 \times 27720$

$= 256 \times 27720 = 7096320$

76. (B) $\sqrt[3]{3^x} = 5^{1/4}$

$3^{x/3} = 5^{1/4}$ .....(i)

and $\sqrt[4]{5^y} = \sqrt{3}$

$5^{y/4} = 3^{1/2}$

$5 = 3^{\frac{1}{2}\times\frac{4}{y}}$ .....(ii)

Putting the value of 5 from equation (ii) in equation (i), we have

$3^{x/3} = \left(3^{\frac{1}{2}\times\frac{4}{y}}\right)^{1/4}$

$\frac{x}{3} = \frac{1}{2}\times\frac{4}{y}\times\frac{1}{4}$

$2xy = 3$

77. (A) Here,

P = 20, Q = 15 km, R = 10 km

x = 5 km/hr, y = 3 km/h, z = 2 km/h

Required average speed = $\frac{P+Q+R}{\frac{P}{x}+\frac{Q}{y}+\frac{R}{z}} = \frac{20+15+10}{\frac{20}{5}+\frac{15}{3}+\frac{10}{3}}$

$= \frac{45}{4+5+5} = \frac{45}{14} = 3\frac{3}{14}$ km/h

78. (A) Let the quantity of milk replaced be x.

ATQ,

$\frac{40}{100}(1-x)+\frac{19}{100}\times x = \frac{26}{100}\times 1$

$40 - 40x + 19x = 26$

$21x = 14$

$x = \frac{2}{3}$

79. (A) Rate × consumption = Expenditure

10/kg × 10 kg = 100

32% increment 10% increment

13.20 kg × x = 110

$$x = \frac{110}{13.2} = \frac{100}{12} = 8\frac{1}{3} \text{ kg}$$

80. (A) C. P. = ₹ 100, M.P. = ₹ 120

$$D = \frac{15}{100} \times 120 = 18$$

S.P. = ₹ 102

$$P\% = \frac{P}{C.P.} \times 100 = \frac{2}{100} \times 100 = 2\%$$

81. (B) $M_1D_1W_2 = M_2D_2W_1$

$$M_2 = \frac{M_1D_1W_2}{D_2W_1} = \frac{45 \times 200 \times 7.5}{150 \times 4.5} = 100$$

Extra men = 100 – 45 = 55 men

82. (B) $\frac{1}{x+1} + \frac{2}{y+2} + \frac{1009}{z+1009} = 1$

$$\frac{1}{x+1} - 1 + \frac{2}{y+2} - 1 + \frac{1009}{z+1009} - 1 = 1$$

$$-\frac{x}{1+x} - \frac{y}{y+2} - \frac{z}{z+1009} = -2$$

$$\frac{x}{1+x} + \frac{y}{y+2} + \frac{z}{z+1009} = 2$$

83. (A)

Raju	Rinku	Ram
6 days	12 days	25 days
4	2	1
24		

Efficiency = 4 + 2 + 1 = 7 units/day

Required time = $\frac{24}{7} = 3\frac{3}{7}$ days

84. (B)

14% (P)		6%(L)
	4% (L)	
2		18
1	:	9

On 14% profit = $\frac{1}{10} \times 50 = 5$ kg

On 6% loss = $\frac{9}{10} \times 50 = 45$ kg

85. (A) Required time = $\frac{8\times16}{16-8}$ = 16 hours

86. (B) $\cos(\alpha+\beta) = 0 = \cos 90°$

$\alpha + \beta = 90°$

$\alpha = 90° - \beta$

Now, $\alpha - \beta = 90° - 2\beta$

$\sin(\alpha-\beta) = \sin(90° - 2\beta) = \cos 2\beta$

87. (A) P = ₹ 16000

r = 5% per annum i.e. $\frac{5}{2}$% (half yearly),

T = $1\frac{1}{2}$ years = 3 half years

Amount = $16000 \times \left(1+\frac{5}{2\times100}\right)^3$

= $16000 \times \left(\frac{41}{40}\right)^3 = 16000 \times \frac{68921}{64000} = ₹17230\frac{1}{4}$

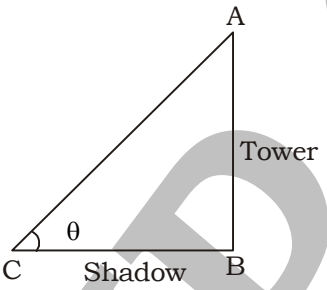
88. (A) For no solution

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{1}{3} = \frac{2}{k} \neq \frac{5}{-15}$

$k = 6$

89. (C)



$\tan\theta = \frac{AB}{BC} = \frac{x}{\frac{x}{\sqrt{3}}} = \sqrt{3} = \tan 60°$

$\theta = 60°$

90. (C) $x = \sqrt{7\sqrt{7\sqrt{7}.....}}$

$x = \sqrt{7x}$

$x^2 = 7x$

$x(x-7) = 0$

$x = 7$

$7 = \left((7)^3\right)^{y-1}$

7^{3y-3}

$3y - 3 = 1$

$y = \frac{4}{3}$

91. (B) $a = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} + \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} + \frac{1}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}}$

$= \frac{2+\sqrt{3}}{4-3} + \frac{3+\sqrt{8}}{9-8} + \frac{4+\sqrt{15}}{16-15}$

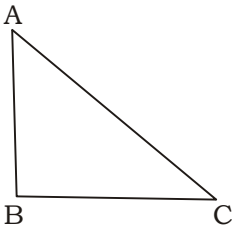
$= 2 + \sqrt{3} + 3 + \sqrt{8} + 4 + \sqrt{15}$

$= 9 + \sqrt{3} + \sqrt{8} + \sqrt{15}$

$= 9 + 1.732 + 2.828 + 3.87 = 17.43 < 18$

92. (D) Power of x must be a positive integer.

93. (A)



AB = BC = x units

AC = $\sqrt{BC^2 + AB^2} = \sqrt{x^2 + x^2}$

Now, Perimeter = $2p$

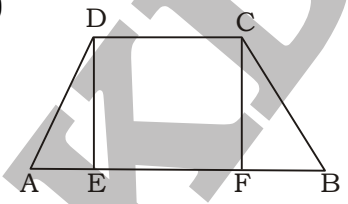
$2x + \sqrt{2}\,x = 2p$

$x = \left(2 - \sqrt{2}\right)p$

Area of $\triangle$ABC = $\frac{1}{2}x^2 = \frac{1}{2}\left(2-\sqrt{2}\right)^2 p^2$

$= \left(3 - 2\sqrt{2}\right)p^2$ sq. units

94. (A)



In $\triangle$ABC, $\angle$B is acute angle

$AC^2 = BC^2 + AB^2 - 2AB \times AE$

In $\triangle$ABD $\angle$A is acute angle

$BD^2 = AD^2 + AB^2 - 2AB \,.\, AF$

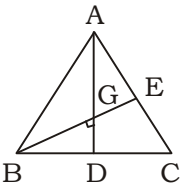
Then,

$AC^2 + BD^2 = BC^2 + AD^2 + 2AB\,(AB - BE - AF)$

$= BC^2 + AD^2 + 2AB.EF$

$= BC^2 + AD^2 + 2AB.CD$

95. (C)



AD = 9 cm

GD = $\frac{1}{3}$ × 9 = 3 cm

BE = 6 cm Þ BG = $\frac{2}{3}$ × 6 = 4 cm

BD = $\sqrt{3^2+4^2}$ = 5 cm

96. (D) Total employees in marketing = 3600 × $\frac{18}{100}$

Number of Male = $\frac{7}{12}$ × 36 × 18 = 378

97. (C) Total number of employees working in HR department = 3600 × $\frac{14}{100}$

Total number of women employees working in HR department = $\frac{3}{4}$ × 36 × 14

Required ratio = 27 × 14 : 36 × 14 = 3 : 4

98. (A) Total male employees in production department = $\frac{11}{12}$ × 3600 × $\frac{28}{100}$ = 924

Total male employees in Accounts department = $\frac{2}{9}\times 3600\times\frac{17}{100}$ = 136

Total male employees in production and account department = 924 + 136 = 1060

99. (B) The number of women working in the IT and HR departments = $\frac{4}{9}$ × 3600 × $\frac{23}{100}$ + 378 = 746

Required percentage = $\frac{746}{3600}$ × 100 = 20.72%

100. (D) The number of women employees working in the marketing and accounts department

= $\frac{5}{12}$ × 3600 × $\frac{18}{100}$ + $\frac{7}{9}$ × 3600 × $\frac{17}{100}$

15 × 18 + 28 × 17 = 746

The number of male employees working in the marketing and accounts departments

= 3600 × $\frac{35}{100}$ – 746 = 1260 - 746 = 514

Required percentage = $\frac{746}{514}\times 100$ = 145.13%

QUANTITATIVE ABILITY - 81 (ANSWER KEY)

1.	(C)	26.	(C)	51.	(C)	76.	(B)
2.	(D)	27.	(D)	52.	(A)	77.	(A)
3.	(D)	28.	(D)	53.	(D)	78.	(A)
4.	(A)	29.	(A)	54.	(C)	79.	(A)
5.	(C)	30.	(A)	55.	(A)	80.	(A)
6.	(A)	31.	(C)	56.	(A)	81.	(B)
7.	(A)	32.	(B)	57.	(C)	82.	(B)
8.	(B)	33.	(C)	58.	(D)	83.	(A)
9.	(B)	34.	(D)	59.	(D)	84.	(B)
10.	(D)	35.	(D)	60.	(C)	85.	(A)
11.	(B)	36.	(C)	61.	(C)	86.	(B)
12.	(D)	37.	(C)	62.	(C)	87.	(A)
13.	(C)	38.	(A)	63.	(D)	88.	(A)
14.	(A)	39.	(C)	64.	(C)	89.	(C)
15.	(D)	40.	(C)	65.	(D)	90.	(C)
16.	(B)	41.	(B)	66.	(A)	91.	(B)
17.	(C)	42.	(A)	67.	(C)	92.	(D)
18.	(B)	43.	(D)	68.	(A)	93.	(A)
19.	(B)	44.	(C)	69.	(D)	94.	(A)
20.	(C)	45.	(D)	70.	(A)	95.	(C)
21.	(A)	46.	(C)	71.	(B)	96.	(D)
22.	(B)	47.	(B)	72.	(C)	97.	(C)
23.	(A)	48.	(C)	73.	(D)	98.	(A)
24.	(A)	49.	(D)	74.	(A)	99.	(B)
25.	(D)	50.	(C)	75.	(A)	100.	(D)