NDA (MATHS) MOCK TEST - 59 (SOLUTION)

1. (A) The equation of family of rectangular hyperbole is
$xy = c^2$
On differentiating, we get
$$y + x\frac{dy}{dx} = 0$$
$\therefore$ Degree and order of differential equation are 1 and 1 respectively.

2. (C) $\because f(x) = x^2 - 2x$
On differentiating w.r.t. x, we get
$f'(x) = 2x - 2$
$f(x)$ is increasing, if $f'(x) > 0$
$2x - 2 > 0$
$x > 1$ only.

3. (B) Let $I = \int_0^1 x(1-x)^n\,dx$
Put $1 - x = t$ and $dx = -dt$
$$I = -\int_1^0 (1-t)t^n dt$$
$$= \int_0^1 (t^n - t^{n+1})\,dt$$
$$= \left[\frac{t^{n+1}}{n+1} - \frac{t^{n+2}}{n+1}\right]_0^1$$
$$= \frac{1}{n+1} - \frac{1}{n+2}$$
$$= \frac{1}{(n+1)(n+2)}$$

4. (B) By Rolle's theorem, between a and b. There exists at least one root of the polynomial equation $f'(x) = 0$.

5. (B) $3^x + 3^y = 3^{x+y}$
On differentiating w.r.t. x, we get
$$3^x \log 3 + 3^y \log 3\,\frac{dy}{dx} = 3^{(x+y)}\log 3\left(1 + \frac{dy}{dx}\right)$$
$$\Rightarrow 3^x + 3^y\frac{dy}{dx} = 3^{x+y} + 3^{(x+y)}\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx}(-3^{x+y} + 3^y) = 3^{x+y} - 3^x$$
$$\Rightarrow \frac{dy}{dx} = \frac{3^x(3^y-1)}{3^y(1-3^x)} = \frac{3^{x-y}(3^y-1)}{(1-3^x)}$$

6. (D) Let $I = \int \sec x^\circ\,dx$
$$= \int \sec\frac{\pi x}{180^\circ}dx \quad \left(\because 1^\circ = \frac{\pi}{180}\right)$$
Put $\frac{\pi x}{180} = t$
$$\Rightarrow dx = \frac{180^\circ}{\pi}dt$$
$$\therefore I = \int \sec t\,dt\,\frac{180^\circ}{\pi}$$
$$= \frac{180^\circ}{\pi}\log\tan\left(\frac{\pi}{4} + \frac{t}{2}\right) + C$$
$$= \frac{180^\circ}{\pi}\log\tan\left(\frac{\pi}{4} + \frac{\pi x}{360^\circ}\right) + C$$

7. (C) $P(x) = -3500 + (400 - x)x$ [given]
On differentiating w.r.t. x, we get
$P'(x) = 400 - 2x$
Put $P'(x) = 0$, for maxima or minima
$400 - 2x = 0$
$\Rightarrow x = 200$
Now, $P''(x) = -2x$
$\Rightarrow P''(200) = -2 < 0$
$\therefore P(x)$ is maximum at $x = 200$
Hence, required number of items = 200.

8. (B) $\because s = 64t - 16t^2$ [Given]
On differentiating w.r.t. x, we get
$$\frac{dx}{dt} = 64 - 32t$$
Put $\frac{dx}{dt} = 0$, for maximum height
$64 - 32t = 0$
$\Rightarrow t = 2$
$$\therefore \left(\frac{d^2x}{dt^2}\right)_{t=2} = 32 < 0 \quad \text{(maximum)}$$
Hence, the required time = 2 s.

9. (D) $\because f(x) = 3x^2 + 6x - 9$
On differentiating w.r.t. x, we get
$f'(x) = 6x + 6$
For a decreasing function.
$f'(x) < 0 \Rightarrow 6x + 6 < 0 \Rightarrow x < -1$
$\therefore f(x)$ is decreasing in $(-\infty, -1)$

10. (A) $\because f(x) = \sin^2 x^2$

$f'(x) = 2\sin x^2 . \cos x^2 \frac{d}{dx}(x^2)$

$\Rightarrow f'(x) = 4x \sin x^2 \cos x^2$

11. (A) $\because f(x) = \cos x, g(x) = \log x$

$y = gof(x)$

$= g\{f(x)\} = g\{\cos x\}$

$= \log(\cos x)$

$\therefore \left(\frac{dy}{dx}\right)_{at\, x=0} = \tan 0 = 0$

12. (D) $\because f(x) = \begin{cases} 3x-4 & 0 \le x \le 2 \\ 2x+\lambda & 2 < x \le 3 \end{cases}$

Also, $f(x)$ is continuous at $x = 2$.

$\therefore \lim_{x\to 2} f(x) = f(2)$

$\Rightarrow \lim_{x\to 2}(2x+\lambda) = 6-4$

$\Rightarrow \lim_{h\to 0} 2(2-h)+\lambda = 2$

$4 + \lambda \Rightarrow \lambda = -2$

13. (B) Since, slope of line $x\cos\theta + y\sin\theta = 2$ is $-\cot\theta$ and slope of the line $x - y = 3$ is 1. Also, these lines are perpendicular to each other.

$\therefore (-\cot\theta)(1) = -1$

$\Rightarrow \cot\theta = 1 = \cot\frac{\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{4}$

14. (A) Since, x-axis is a tangent to the given circle, it means the circle touches the x-axis.

$\therefore 2\sqrt{g^2-k} = 0$

$\Rightarrow g^2 = k$

15. (B) We know that, the sum of focal radii of any point on an ellipse is equal to length of major axis, *i.e.*, sum of focal radii

$= (a + x) + (a - x)$

$= 2a =$ Major axis

16. (B) Equation of the first order containing one arbitrary parameter and passing through a given points represents a straight line.

17. (B) The equation of line perpendicular to the given line

$x + y - 11 = 0$ ... (i)

is $-x + y + \lambda = 0$ ... (ii)

This equation passes thorugh (2, 3).

$\therefore -2 + 3 + \lambda = 0$

$\lambda = -1$

From Eqn (ii),

$-x + y - 1 = 0$

$\Rightarrow y = x + 1$

$\therefore$ From Eq. (i),

$x + x + 1 - 11 = 0$

$\Rightarrow 2x = 10$

$\Rightarrow x = 5$ and $y = 5 + 1 = 6$

Hence, coordinates of foot of perpendicular from (2, 3) to given line is (5, 6).

18. (A) We know that, the equation of x-axis is $y = 0$. Thus, only statement I is correct.

19. (B) Let θ be the angle between given planes, then

$\cos\theta = \left|\frac{2\times1+1\times(-1)+1\times2}{\sqrt{4+1+1}\sqrt{1+1+4}}\right|$

$= \frac{3}{6} = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3}$

20. (B) The equation of the plane passing through x-axis is $x = a$.

This also passes through (1, 2, 3)

$\therefore x = 1$ $[\because a = 1]$

Which is the required equation of plane.

21. (A) Let the coordinates of the points A, B, C and D be (1, 3, 4), (–1, 6, 10), (–7, 4, 7) and (–5, 1, 1) respectively.

$\therefore AB = \sqrt{(-1-1)^2+(6-3)^2+(10-4)^2}$

$= \sqrt{4+9+36} = 7$

$BC = \sqrt{(-7+1)^2+(4-6)^2+(7-10)^2}$

$= \sqrt{36+4+9} = 7$

$DA = \sqrt{(1+5)^2+(3-1)^2+(4-1)^2}$

$= \sqrt{36+4+9} = 7$

$AC = \sqrt{(-7-1)^2+(4-3)^2+(7-4)^2}$

$= \sqrt{64+1+9} = \sqrt{74}$ and,

$CD = \sqrt{(-5+7)^2+(1-4)^2+(1-7)^2}$

$= \sqrt{4+9+36} = 7$

BD = $\sqrt{(-5+1)^2+(1-6)^2+(1-10)^2}$

= $\sqrt{16+25+81}=\sqrt{122}$

$\because$ AB = BC = CD = DA

But BD $\neq$ AC

$\therefore$ Points A, B, C and D form a rhombus.

22.(C) We know that the number of planes passing through three non-collinear points is 1.

23. (A) Given that,

$x + z = 0$, $y = 0$ and $20x = 15y = 12z$

$\Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ = and $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

Let θ be the angle between both lines.

Then, $\cos\theta = \left|\frac{(1)(3)+(0)(4)+(-1)(5)}{\sqrt{1+0+1}\sqrt{9+16+25}}\right|$

$\Rightarrow \cos\theta = \left|\frac{3+0-5}{\sqrt{2}\sqrt{50}}\right| = \left|\frac{2}{\sqrt{2}\cdot 5\sqrt{2}}\right| = \frac{1}{5}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{5}\right)$

24. (D) Latus rectum of an ellipse = $\frac{2b^2}{a}$,

and minor axis = 2b

$\therefore b = \frac{2b^2}{a} \Rightarrow a = 2b$ [ATQ]

Also, $e = \sqrt{1-\frac{b^2}{a^2}}$

$= \sqrt{1-\frac{b^2}{4b^2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

25. (D) A = {{1, 2}} = { {1}, {2}, {3} }

$\Rightarrow$ {1, 2} $\in$ A

26. (C) The relation is defined as xRy, iff if 3x + 4y = 5

if we take, (x, y) = $\left(1,\frac{1}{2}\right)$ and $\left(\frac{2}{3},\frac{3}{4}\right)$

then these pairs are satisfied by the given relation

$1\ R\frac{1}{2} \Leftrightarrow 3.1 + 4y = 5$

and $\frac{2}{3}\ R\ \frac{3}{4} \Leftrightarrow \frac{2}{3}.3 + 4.\frac{3}{4} = 5$

27. (C) $\because$ fog (x) = 25

$\Rightarrow$ f [g (x)] = 25

$\Rightarrow$ f [x^2 + 7] = 25

$\Rightarrow$ 2 (x^2 + 7) + 7 = 25

$\Rightarrow$ $2x^2 = 2$

$\Rightarrow$ $x^2 = 4$

$\therefore x = \pm \sqrt{2}$

28. (C) $4 + 5\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)^{334} + 3\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)^{365}$

$= 4 + 5(\omega)^{334} + 3(\omega)^{365}$

$= 4 + 5(\omega^3)^{111}\omega + 3(\omega^3)^{121}\omega^2$

$= 4 + 5\omega + 3\omega^2$

$= 3\omega^2 + 3\omega + 3 + 2\omega + 1$

$= 3(\omega^2 + \omega + 1) + 2\omega + 1$

$= 2\omega + 1$

$= 2\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right) + 1$

$= -1 + i\sqrt{3} + 1$

$= i\sqrt{3}$

29. (C) In general, rth term of the the given expression

$T_r = (r+1)\left(r+\frac{1}{\omega}\right)\left(r+\frac{1}{\omega^2}\right)$

$= r^3 + r^2\left(\frac{1}{\omega^2}+\frac{1}{\omega}+1\right) + r\left(1+\frac{1}{\omega^2}+\frac{1}{\omega}\right) + 1$

$= r^3 + r^2(\omega + \omega^2 + 1) + r(\omega + \omega^2 + 1) + 1$

$= r^3 + 1$ $(\because 1 + \omega + \omega^2 = 0)$

$\therefore S_n = \sum_{r=1}^{n}(r^3+1)$

$= \left\{\frac{n(n+1)}{2}\right\}^2 + n = \frac{n^2(n+1)^2}{4} + n$

$= \frac{n^2(n+1)^2+4n}{4}$

30. (C)

2	83	
2	41	1
2	20	1
2	10	0
2	5	0
2	2	1
	1	0

$\therefore (83)_{10} = (1010011)_2$

Which is required binary form.

31. (D) $(0.101)_2 = 2^{-1} \times 1 + 2^{-2} \times 0 + 2^{-3} \times 1$

$= \frac{1}{2} + 0 + \frac{1}{8} = \frac{5}{8}$

and $(0.011)_2 = 0 \times 2^{-1} + 2^{-2} + 1 \times 2^{-3} \times 1$

$= 0 + \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

Also $(11)_2 = 1 \times 2^1 + 1 \times 2^0 = 3$

and $(01)_2 = 0 \times 2^1 + 1\ 2^0 = 1$

$\therefore \frac{(0.101)_2^{(11)_2} + (0.011)_2^{(11)_2}}{(0.101)_2^{(10)_2} - (0.101)_2^{(01)_2}(0.011)_2^{(01)_2} + (0.011)_2^{(10)_2}}$

$= \frac{\left(\frac{5}{8}\right)^3 + \left(\frac{3}{8}\right)^3}{\left(\frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)\left(\frac{3}{8}\right) + \left(\frac{3}{8}\right)^2}$

$= \frac{5}{8} + \frac{3}{8} + \frac{8}{8} = 1 = (1)_2$

32. (B) $T_m = S_m - S_{m-1}$

$164 = 3m^2 + 5m - [\ 3\ (m-1)^2\ + 5\ (m-1)]$

$= 3m^2 + 5m - [\ 3\ (m^2 - 2m + 1)\ + 5\ m - 5]$

$= 3m^2 + 5m - \ 3\ m^2 + 6m - 3\ - 5\ m + 5$

$164 = 6m + 2$

$\Rightarrow\ m = \frac{164-2}{6} = 27$

33.(C) Let $S_n = \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{14} + \ldots\ldots + a_n$

also, $S_n\ = \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \ldots + a_{n-1} + a_n$

$\Rightarrow\ 0\ = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots a_n$

$\Rightarrow\ a_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ n terms

$a_n = \frac{\frac{1}{2}\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}} = 1 - 2^{-n}$

$\Rightarrow\ a_n = \frac{2^n - 1}{2^n}$

$S_n = \sum 1 - \sum 2^{-n}$

$= n - \{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ n terms$\}$

$= n - \frac{\frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}}$

$= n - (1 - 2^{-n}) = n - 1 + 2^{-n}$

34. (A) $\because$ a, b, c are in GP.

$\Rightarrow b^2\ = ac$

Now $\frac{1}{a^2 - b^2} + \frac{1}{b^2}$

$\Rightarrow \frac{b^2 + a^2 - b^2}{b^2(a^2 - b^2)}$

$\Rightarrow \frac{a^2}{b^2(a^2 - b^2)} = \frac{\left(\frac{b^2}{c}\right)^2}{b^2\left[\left(\frac{b^2}{c}\right)^2 - b^2\right]}$

$= \frac{\frac{b^4}{c^2}}{\frac{b^2(b^4 - b^2c^2)}{c^2}} = \frac{b^4}{b^2 \times b^2\ (b^2 - c^2)}$

$= \frac{1}{b^2 - c^2}$

35. (C) $\because \frac{a+b}{2} = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$

$\Rightarrow\ (a + b)\ (a^n + b^n) = 2\ (a^{n+1} + b^{n+1})$

$\Rightarrow\ a^{n+1} + ab^n + a^nb + b^{n+1} = 2a^{n+1} + 2b^{n+1}$

$\Rightarrow ab^n + a^nb = a^{n+1} + b^{n+1}$

$\Rightarrow ab^n - b^{n+1} = a^{n+1} - a^nb$

$\Rightarrow\ b^n\ (a - b) = a^n\ (a - b)$

$\Rightarrow \left(\frac{a}{b}\right)^n = 1 = \left(\frac{a}{b}\right)^0$

$\Rightarrow\ \ n = 0$

36. (A) $\because\ S_n = 3n^2 + 5n$

$\therefore\ T_n = S_n - S_{n-1} = (3n^2 + 5n)\ - [3\ (n-1)^2 + 5\ (n-1)$

$= 3n^2 + 5n - 3n^2 - 3 + 6n - 5n + 5$

$= (6n + 2)$

The nth term is a linear function in n. Hence, sequence must be an AP.

37. (D) Given, $T_n = 5456 \Rightarrow\ n = 5456$

$\Rightarrow 6n = 5454 \Rightarrow\ n = 909$

$\therefore$ The number 5456 is the (909)th term.

38. (B) $T_1^2 + T_2^2 + T_3^2 = (8)^2 + (14)^2 + (20)^2$
$= 64 + 196 + 400 = 660$

39. (B) Since, α, β are the roots of $x^2 + x\sqrt{\alpha} + \beta = 0$, then $\alpha + \beta = -\sqrt{\alpha}$ and
$\alpha\beta = \beta$
$\Rightarrow \alpha = 1, \beta = -2$

40. (B) Now, $\alpha + 1 = 1 + 1 = 2$,
$\beta + 1 = -2 + 1 = -1$
Their sum = 2 + (-1) = 1
Their product = 2 (-1) = -2
$\therefore$ Required equation
$x^2 - (1)x + (-2) = 0 \Rightarrow x^2 - x - 2 = 0$

41. (A) First we arrange the four letters G, L, M and Y in the alternate position = 4!
Now, rest of letters 0, 0 arranged in 5 alternate positions = $^5C_2$
$\therefore$ Required number of ways = 4! × $^5C_2$
$= 24 \times \frac{5\times4}{2}$
= 24 × 10 = 240

42. (B) Required number of ways= $2^6 - 1 = 64-1$
= 63

43. (B) $T_{r+1} = {}^{10}C_r\, x^{\frac{10-r}{2}} \times \frac{1}{3^{\frac{10-r}{2}}} \times \frac{3^r}{2^r.r^{2r}}$

$= {}^{10}C_r \frac{3^r}{3^{\frac{10-r}{2}}} \times \frac{1}{2^r} \times x^{\frac{10-r}{2}-2r}$

$\Rightarrow x^{\frac{10-r}{2}-2r} = x^0$
10 – r – 4r = 0   r = 2

$\therefore$ coefficient = ${}^{10}C_2 \frac{3^2}{3^4} \times \frac{1}{2^2}$

$= 5 \times \frac{1}{4}$
= 5/4

44. (C) Number of terms in
x + y or $(a + 1)^6 - (a - 1)^6$ is equal to $\frac{6}{2} + 1 = 4$

45. (C) Number of terms in x - y or $(a + 1)^6 - (a-1)^6$ is equal to $\frac{6}{2} = 3$.

46. (B) $x - y = (a + 1)^6 - (a - 1)^6$
$= 2\,({}^6C_1a^5 + {}^6c_3a^3 + {}^6C_5a)$
$= 2\,(6a^4 + {}^6C_3a^2 + {}^6C_5)\,a$
$= 2\,(6 \times 4 + 20 \times 2 + 6)\sqrt{2}$
$= 2\sqrt{2}\,(24 + 40 + 6) = 140\sqrt{2}$

47. (D) Let T_{r+1} be the term independent of x in $\left(\frac{3x^{-2}}{2} - \frac{1}{3x}\right)^9$

$\therefore T_{r+1} = {}^9C_r \left(\frac{3x^{-2}}{2}\right)^{9-r} \left(-\frac{1}{3x}\right)^r$

$= (-1)^r\, {}^9C_r \left(\frac{3}{2}\right)^{9-r} . \frac{1}{3^r} x^{-18+2r-r}$

For coefficient of x^0, x^{-1} and x^{-2},
we get $-18 + r = 0$, $-18 + r = -1$
and $-18 + r = -2$
which is not possible
Thus, no such term exists in the expansion of given expression.

48. (B) $\log_x a + \log_x c = 2 \log_x b$
$\Rightarrow ac = b^2$
$\Rightarrow$ a, b, c are in GP

49. (A) Given, $X + Y = 15$
The total number of ordered pairs
= (5,10), (6,9), (7,8), (8,7), (9,6), (10,5)
$\therefore n(E) = 6$
In each above pairs exactly one is even number.
$\therefore n(E) = 6$

$\therefore$ Required probability = $\frac{n(E)}{n(S)} = \frac{6}{6} = 1$

50. (A) Since, $(2x + 2)^2 = x(3x + 3)$
$\Rightarrow x^2 + 5x + 4 = 0$
$\Rightarrow x = -1, -4$
Now, first term $a = x$

Second term, $ar = 2(x + 1) \Rightarrow r = \frac{2(x+1)}{x}$

$\therefore$ Fourth term = $ar^3 = x\left(\frac{2(x+1)}{x}\right)^3$

Put $x = -4$, we get

$T_4 = -4\left(\frac{2(-4+1)}{-4}\right)^3 = -4\times\left(\frac{3}{2}\right)^3 = -\frac{27}{2}$

51. (C) Correlation between two variables is said to be perfect, if when one variable increases, the other also increases proportionally.

52. (C) Both statements are true.

53. (C) $\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cap B) = \frac{1}{3} + \frac{3}{4} - \frac{11}{12} = \frac{1}{6}$$

$$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/6}{1/3} = \frac{1}{2}$$

54. (A) Given,

$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$

$\Rightarrow 3x^2 - 2(b + a + c)x + ab + bc + ca = 0$

Now, D = $\sqrt{4(a+b+c)^2 - 12(ab+bc+ca)}$

$= 2\sqrt{a^2 + b^2 + c^2 - ab - bc - ca}$

$= 2\sqrt{\frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\}}$

> 0

Hence, roots are always real.

55. (B) $(\log_x x)(\log_3 2x)(\log_{2x} y) = \log_x x^2$

$\Rightarrow 1(\log_3 2x)(\log_{2x} y) = 2 \quad (\therefore \log_x x = 1)$

$\Rightarrow \log_3 y = 2$

$\Rightarrow y = 3^2$

$\Rightarrow y = 9$

56. (D) Given, $\log_5 k \log_k x = 3$

$\Rightarrow \log_5 x = 3$

$\Rightarrow x = 5^3$

$\Rightarrow x = 125$

57.(B) Given, series can be rewritten as

$20, \frac{77}{4}, \frac{37}{2}, \frac{71}{4}, \ldots\ldots$

This is an AP series,

Here, $a = 20$, $d = -\frac{3}{4}$

$T_n = a + (n-1)d$

$= 20 + (n-1)\left(-\frac{3}{4}\right)$

$= \frac{83}{4} - \frac{3}{4}n$

For first negative term, $T_n < 0$

$\Rightarrow \frac{83}{4} - \frac{3}{4}n < 0$

$\Rightarrow 83 < 3n$

$\Rightarrow n > \frac{83}{3}$

So, n should be 28.

Hence, 28 th term is the first negative term.

58. (B) $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$$\therefore \cot^{-1}x = \cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \sin^{-1}x + \cos^{-1}\frac{1}{\sqrt{5}} = \frac{\pi}{2} = \sin^{-1}x + \cos^{-1}x$$

$$\left(\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}\right)$$

On comparing

$\therefore x = \frac{1}{\sqrt{5}}$

59. (C) Let given equations have common root α.

Then, $\alpha^2 + m\alpha + 1 = 0$ and $\alpha^2 + \alpha + m = 0$

$$\Rightarrow \frac{\alpha}{m^2 - 1} = \frac{\alpha}{1-m} = \frac{1}{1-m}$$

$$\Rightarrow \frac{\alpha}{1-m} = \frac{1}{1-m} \Rightarrow \alpha = 1$$

Also, $\frac{\alpha^2}{m^2 - 1} = \frac{1}{1-m}$

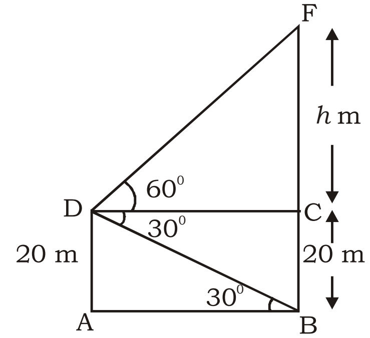
$\Rightarrow 1 - m = m^2 - 1$

$\Rightarrow m^2 + m - 2 = 0$

$\Rightarrow (m + 2)(m - 2) = 0$

$\Rightarrow m = 1$ and -2

60.(D) In Δ ABD, $\tan 30^0 = \frac{AD}{AB} = \frac{20}{AB}$



$\Rightarrow AB = 20\sqrt{3}$ m.

In ΔDCF, $\tan 60^0 = \frac{h}{DC} = \sqrt{3}$

$\Rightarrow h = \sqrt{3}.20\sqrt{3} \quad (AB = DC = 20\sqrt{3})$

$\Rightarrow h = 60$ m [From Eq. (i)]

$\therefore$ Height of tower, $BF = 60 + 20 = 80$m

($\because BF = h + 20$)

61.(A) $\sqrt{3}\,\text{cosec}20^0 - \sec 20^0 = \dfrac{\sqrt{3}}{\sin 20^0} - \dfrac{1}{\cos 20^0}$

$$= 2\left(\frac{\frac{\sqrt{3}}{2}\cos 20^0 - \frac{1}{2}\sin 20^0}{\sin 20^0 \cos 20^0}\right)$$

$$= 2\left(\frac{\sin 60^0.\cos 20^0 - \cos 60^0.\sin 20^0}{\sin 20^0.\cos 20^0}\right)$$

$$= \frac{2[\sin(60^0 - 20^0)]}{\frac{1}{2}\sin 40^0}$$

$$= \frac{4\sin 40^0}{\sin 40^0} = 4$$

62. (B) a. $\tan 15^0 = \tan(45^0 - 30^0) = \dfrac{\tan 45^0 - \tan 30^0}{1 + \tan 45^0.\tan 30^0}$

$$\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{3 + 1 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

b. $\tan 75^0 = \tan(45^0 + 30^0)$

$$= \frac{\tan 45^0 + \tan 30^0}{1 - \tan 45^0 \tan 30^0}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 2 + \sqrt{3}$$

c. $\tan(105^0) = \tan(60^0 + 45^0)$

$$= \frac{\tan 60^0 + \tan 45^0}{1 - \tan 60^0.\tan 45^0}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$$

63. (D) Given, $a + b = 3(1 + \sqrt{3})$ .... (i)

and $a - b = 3(1 - \sqrt{3})$ .... (ii)

On solving, we get $a = 3$, $b = 3\sqrt{3}$

By using sine rule, $\dfrac{a}{sinA} = \dfrac{b}{\sin B}$

$$\Rightarrow \frac{3}{\sin 30^0} = \frac{3\sqrt{3}}{\sin B}$$

$\Rightarrow \sin B = \sqrt{3} \times \dfrac{1}{2} = \sin 60^0$

$\Rightarrow B = 60^0$

64. (C) Given,

$\therefore\ N_a = \{ax \mid x \in N\}$

$N_{12} = \{12, 24, 36, 48, \ldots\}$

and $N_8 = \{8, 16, 24, \ldots\}$

$\therefore\ N_8 \cap N_{12} = \{24, 48, \ldots\} = N_{24}$

65. (B) $X = \{(4^n - 3n - 1)n \in N\}$

and $Y = \{9(n - 1) \mid n \in N\}$

$\Rightarrow\ X = \{0, 9, 54\ldots\}$

$Y = \{0, 9, 18, 27, 36, 54, \ldots\}$

$\therefore\ X \cup Y = \{0, 9, 18, 27, 36, 54, \ldots\}$

$= Y$

66. (D) The total number of elements common in (A × B) and (B × A) is n^2.

(By property)

67. (D) Given, $z + z^{-1} = 1$

$\Rightarrow\ z^2 - z + 1 = 0$

$\Rightarrow\ z = -\omega, -\omega^2$

when $z = -\omega$

$\therefore\ z^{99} + z^{-99} = (-\omega)^{99} + (-\omega)^{-99} = -1-1=-2$

($\because \omega^3 = -1$)

when $z = -\omega^2$

$\therefore\ z^{99} + z^{-99} = (-\omega^2)^{99} + (-\omega^2)^{99}$

$= -1 -1 = -2$

68. (D) Given, $T_m = a + (m-1)d$

$\Rightarrow\ \dfrac{1}{n} = a + (m-1)d$

and $T_n = a + (n-1)d$

$\Rightarrow\ \dfrac{1}{m} = a + (n-1)d$

On solving Eqs. (i) and (ii)

$a = d = \dfrac{1}{mn}$

$\therefore\ T_{mn} = \dfrac{1}{mn} + (mn-1)\dfrac{1}{mn} = 1$

69. (B) Number of times 3 occurs = (when 3 occurs exactly at one place) + (when 3 occurs exactly at two places) + (when 3 occurs exactly at three places)

$= {}^3C_1 \times 9 \times 9 + {}^3C_2 \times 9 + {}^3C_3$

$= 243 + 27 + 1 = 271$

70. (A) I. The coefficient of middle term in the expansion of

$(1 + x)^8 = {}^8C_4$

and $\left(x + \frac{1}{x}\right)^8 = {}^8C_4$

Hence, it is equal

II. The coefficient of middle term in the expansion of

$(1 + x)^8 = {}^8C_4$

The coefficient of fifth term in the expansion of

$(1 + x)^7 = {}^7C_4$ or ${}^7C_3$

$\therefore {}^8C_4 > {}^7C_4$ or ${}^7C_3$

Hence, only statement I is true and statement II is false.

71. (C) Given, $xy = ae^x + be^{-x}$ .... (i)

Differentiating on both sides w.r.t. x, we get.

$\Rightarrow y + x\frac{dy}{dx} = ae^x - be^{-x}$

Again differntiating w.r.t. x, we get

$\Rightarrow \frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} = ae^x + be^{-x}$

$\Rightarrow x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = xy$ [from Eq. (i)]

Hence, it is of order two and degree one.

72. (B) Given,

$y = (1 + x^{1/4})(1 + x^{1/2})(1 - x^{1/4})$

$= (1 + x^{1/2})[1 - (x^{1/4})^2]$

$= (1 + x^{1/2})(1 - x^{1/2})$

$\Rightarrow y = 1 - x$

On differentiationg w.r.t. x, we get

$\frac{dy}{dx} = -1$

73. (D) The total daily income of workers, including the owner = 10 × 110 = 1100

The total daily income of workers, excluding the owner = 9 × 76 = 684

$\therefore$ Daily income of owner

= 1100 – 684 = ₹416

74. (D) $\because$ A, B, C are non-empty sets such that $A \cap C = \phi$.

$\therefore (A \times B) \cap (C \times B) = (A \cap C) \times B$

$= \phi \times B$

$= \phi$

75. (D) $\because A = \{4n + 2 : n \in N\}$

$= \{6, 10, 14, 18, 22, 26, 30, ...\}$

and $B = \{3n : n \in N\}$

$= \{3, 6, 9, 12, 15, 18, 21, 24, ...\}$

$\therefore A \cap B = \{6, 18, 30, ...\}$

$= \{6 + (n - 1)12]\ n \in N\}$

$= \{12n - 6 |\ n \in N\}$

76. (A) $\because A = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 + \beta^2 & 2\alpha\beta \\ 2\alpha\beta & \alpha^2 + \beta^2 \end{bmatrix}$

Now,

$\Rightarrow \begin{bmatrix} \alpha^2 + \beta^2 & 2\alpha\beta \\ 2\alpha\beta & \alpha^2 + \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 + \beta^2 = 1, \ \alpha\beta = 0$

$\Rightarrow \alpha = 0, \ \beta = 1$, or $\beta = 0, \ \alpha = 1$

77.(C) The sum of the focal distances of a point of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $2a$.

[by property]

78. (B) The given equation can be rewritten as

$$\left(\frac{d^3y}{dx^3}\right)^2 = \left(3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right)^3$$

$\therefore$ Degree of the differential equation is 2.

79. (B) $\sqrt{3} + \frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} + ...$ $\left(\because S_\infty = \frac{a}{1 - r}\right)$

$= \frac{\sqrt{3}}{1 - \frac{1}{3}} = \frac{3\sqrt{3}}{2}$

80. (B) Given equation is

$y\frac{dy}{dx} + x = a$

$\Rightarrow \int y\,dy + \int x\,dx = a\int dx$

$\Rightarrow \frac{y^2}{2} + \frac{x^2}{2} - \alpha x = \frac{c}{2}$

$\Rightarrow x^2 + y^2 - 2\alpha x = c$

This represents a circle whose centre is on the x-axis.

81. (B) $\because$ A $= \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

But $A^2 = B \Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 = 1$ and $\alpha + 1 = 2$

$\Rightarrow \alpha = 1$

82. (C) $\because \frac{{}^{15}P_{n-1}}{{}^{16}P_{n-2}} = \frac{3}{4}$ (given) $\left[\because {}^nP_r = \frac{n!}{(n-r)!}\right]$

$\Rightarrow \frac{15!}{(15-n+1)} \times \frac{(16-n+2)!}{16!} = \frac{3}{4}$

$\Rightarrow \frac{(18-n)!}{16(16-n)!} = \frac{3}{4}$

$\Rightarrow (18 - n)(17 - n) = 12$

$\Rightarrow 306 - 17n - 18n + n^2 = 12$

$\Rightarrow n^2 - 35n + 294 = 0$

$\Rightarrow (n - 14)(n - 21) = 0$

$\Rightarrow n = 14$ $(\because n \neq 21)$

83. (D) $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

(use operations, $C_2 \to C_2 - C_1$; $C_3 \to C_3 - C_1$)

$= \begin{vmatrix} x+1 & 1 & 3 \\ x+3 & 2 & 5 \\ x+7 & 3 & 7 \end{vmatrix}$

$= \begin{vmatrix} x+1 & 1 & 3 \\ 2 & 1 & 2 \\ 4 & 1 & 2 \end{vmatrix}$

(use operations, $R_3 \to R_3 - R_2$; $R_2 \to R_2 - R_1$)

$= (x + 1)(0) - 1(4 - 8) + 3(2 - 4)$

$= 4 - 6 = -2$

84. (C) Given that,

$f(x) = x^3 - 1$

$\Rightarrow f'(x) = 3x^2$

For increasing function $f'(x) > 0$

$\Rightarrow 3x^2 > 0$

$\therefore$ The function will increase, for $x \in$ R, i.e., the function will increase in [–1, 1].

The curve does not have any sign change in the interval (–1, 1).

$\therefore$ $f(x)$ has no root in (–1, 1).

85. (B) Let $f(x) = 2x^3 - 3x^2 - 12x + 5$

$f'(x) = 6x^2 - 6x - 12$

For maximum value of $f(x)$

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x(x - 2) + 1(x - 2) = 0$

$\Rightarrow x = -1$ and 2

$f''(x) = 12x - 6$

At $x = -1$, $f''(-1) = -18 < 0$ (max)

At $x = 2$, $f''(2) = 24 - 6 = 18 > 0$ (min)

So, $f(x)$ has maximum value at $x = -1$. where $x \in [-2, 2]$

86. (B) $I = \int_0^{\pi} \frac{dx}{1 + 2\sin^2 x}$

$= 2\int_0^{\frac{\pi}{2}} \frac{dx}{1 + 2\sin^2 x}$

$\because \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$

If $\{f(2a - x) = f(x)\}$

$I = 2\int_0^{\frac{\pi}{2}} \frac{dx}{1 + 1 - \cos 2x}$

$= 2\int_0^{\frac{\pi}{2}} \frac{dx}{2 - \cos 2x}$

$= 2\int_0^{\frac{\pi}{2}} \frac{dx}{2 - \left(\frac{1 - \tan^2 x}{1 + \tan^2 x}\right)}$

$= 2\int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{1 + 3\tan^2 x}\,dx$

Let $\sqrt{3}\tan x = t \Rightarrow \sqrt{3}\sec^2 x\, dx = dt$

$I = 2\int_0^\infty \frac{dt}{\sqrt{3}(1+t^2)} = \frac{2}{\sqrt{3}}[\tan^{-1} t]_0^\infty$

$I = \frac{2}{\sqrt{3}}[\tan^{-1}(\infty) - \tan^{-1}(0)]$

$I = \frac{2}{\sqrt{3}}\left(\frac{\pi}{2} - 0\right) = \frac{2\pi}{2\sqrt{3}}$

$I = \frac{\pi}{\sqrt{3}}$

87. (A) $\begin{vmatrix} x & 4 & 5 \\ 7 & x & 7 \\ 5 & 8 & x \end{vmatrix} = 0$

Expand with respect to R1

$\Rightarrow x(x^2 - 56) - 4(7x - 35) + 5(56 - 5x) = 0$

$\Rightarrow x^3 - 56x - 28x + 140 + 280 - 25x = 0$

$\Rightarrow x^2 - 109x + 420 = 0$

$\Rightarrow (x - 5)(x - 7)(x + 12) = 0$

$\Rightarrow x = -12$

88. (B) If a, ar, ar^2, ... are in GP, then
According to question,

$a = \frac{1}{3}(ar + ar^2)$

$\Rightarrow 3 = r + r^2$

$\Rightarrow r^2 + r - 3 = 0$

$\Rightarrow r = \frac{-1 \pm \sqrt{1 + 4\times 3}}{2}$

$\Rightarrow r = \frac{-1 \pm \sqrt{13}}{2} = \frac{\sqrt{13} - 1}{2}$ $(\because r > 0)$

89. (B) $(1 + \tan\theta)(1 + \tan\phi) = 2$

$\Rightarrow 1 + \tan\theta + \tan\phi + \tan\theta\tan\phi = 2$

$\Rightarrow \tan\theta + \tan\phi = 1 - \tan\theta\tan\phi$

$\Rightarrow \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = 1$

$\Rightarrow \tan(\theta + \phi) = \tan 45^\circ$

$\Rightarrow (\theta + \phi) = 45^\circ$

90. (C) $\because f(x)$ is an even function.

$\therefore \int_0^\pi f(\cos x)\,dx = 2\int_0^{\frac{\pi}{2}} f(\cos x)\,dx$

$\because \int_0^{2a} f(x) \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(2a - x) = f(x), \text{(even)} \\ 0, & \text{if } f(2a - x) = -f(x), \text{(odd)} \end{cases}$

91. (C) $\because a_1 = k, a_2 = b_1 = 2, b_2 = 1, c_1 = 5, c_2 = 1$
For no solution,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{k}{3} = \frac{2}{1} \Rightarrow k = 6$

92. (D) Required number of ways = 6 × 5 = 30.

93. (D) $\log_{10}\frac{9}{8} - \log_{10}\frac{27}{32} + \log_{10}\frac{3}{4}$

$= \log_{10}\frac{9}{8} \times \frac{32}{27} \times \frac{3}{4}$

$= \log_{10} 1 = 0$

94. (B) $\because a = \frac{1}{4}, -\frac{1}{2}\Big/\frac{1}{4} = -2$

So, the given series forms a G.P.

$\therefore T_n = ar^{n-1}$

$\Rightarrow -128 = \frac{1}{4}(-2)^{n-1}$

$\Rightarrow (-2)^9 = (-2)^{n-1}$

$\Rightarrow 9 = n - 1$

$\Rightarrow n = 10$

95. (C) The possible non-zero positive integer ordered pair (x, y) satisfy the inequality $x + y \le 4$ is (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1).
$\therefore$ The number of required ordered pairs = 6

96. (B) $a = 0111 = 2^4 \times 0 + 2^3 \times 0 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 1$
$= 4 + 2 + 1 = 7$

$b = 01110 = 2^4 \times 0 + 2^3 \times 1 + 2^2 \times 1 \times 2^1 \times 1 + 2^0 \times 0$
$= 8 + 4 + 2 = 14$

$\frac{b}{a} = \frac{14}{7} = 2$

97. (C) Given, the distance between the points (7, 1, –3) and (4, 5, λ)
= 13

$\Rightarrow \sqrt{(4-7)^2 (5-1)^2 + (\lambda + 3)^2} = 13$

$\Rightarrow \sqrt{(3)^2 + (4)^2 + (\lambda + 3)^2} = 13$

$\Rightarrow \sqrt{9 + 16 + (\lambda + 3)^2} = 13$

$\Rightarrow \sqrt{25 + (\lambda + 3)^2} = 13$

On squaring both sides we get

$\Rightarrow 25 + (\lambda + 3)^2 = 169$

$\Rightarrow 25 + \lambda^2 + 9 + 6\lambda - 169 = 0$

$\Rightarrow \lambda^2 + 6\lambda - 135 = 0$

$\Rightarrow \lambda^2 + 15\lambda - 9\lambda - 135 = 0$

$\Rightarrow \lambda(\lambda + 15) - 9(\lambda + 15) = 0$

$\Rightarrow (\lambda + 15)(\lambda - 9) = 0$

$\Rightarrow \lambda = 9, -15$

98. (A) In ΔAOP,

$\sin\alpha = \frac{z}{OP} = \frac{z}{r}$

$\Rightarrow z = r\sin\alpha$

Again in ΔAOP,

$\cos\alpha = \frac{x}{OP} = \frac{x}{r}$

$\Rightarrow x = r\cos\alpha$ and the y-cordinate in xz-plane in always zero.

$\therefore$ The coordinate of P in xz-plane is $(r\cos\alpha, 0, r\sin\alpha)$.

99. (A) The point (0, 2, 0) lie on yz-plane corre sponding to the point (1, 2, 0). The distance (1, 2, 0) and (0, 2, 0)

$= \sqrt{(1-0)^2 + (2-2)^2 + (0-0)^2}$

$= \sqrt{1+0+0} = 1$

100. (A) If α, β and γ are the angles that a line makes with the coordinates axes.

$\therefore l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

$\because l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ... (i)

Here, $\alpha = \beta = \gamma$ (given)

$\therefore$ From Eq. (i),

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

(since, direction cosines of a line which is equally inclined to the positive directions of the axis so we take only positive sign)

$\therefore \cos\alpha = \cos\beta = \cos\gamma = \frac{1}{\sqrt{3}}$

or, $l = m = n = \frac{1}{\sqrt{3}}$

Hence, the required direction cosines are $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$.

101. (A) Given lives are,

$\frac{x-2}{1} = \frac{y+1}{-2} = \frac{z+2}{1}$

$\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$

$\frac{x-1}{1} = \frac{2\left(y+\frac{3}{2}\right)}{3} = \frac{z+5}{2}$

$\frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{3}{2}} = \frac{z+5}{2}$

If θ be the acute angle between lines (i) and (ii), then

$\cos\theta = \left| \frac{1\times1 + (-2)\left(\frac{3}{2}\right) + 1(2)}{\sqrt{1+(-2)^2+1^2}\sqrt{1^2+\left(\frac{3}{2}\right)^2+2^2}} \right|$

$= \left| \frac{1 + (-3) + 2}{\sqrt{1+4+1}\sqrt{1+\frac{9}{4}+4}} \right|$

$= \left| \frac{0}{\sqrt{6}\sqrt{\frac{29}{4}}} \right| = 0$

$\therefore \cos\theta = 0$

$\theta = \cos^{-1}(0) = \frac{\pi}{2}$

102. (C) The equation of any plane parallel to the plane $3x + 4y - 5z = 0$ may be taken as

$3x + 4y - 5z + k = 0$ ... (i)

If plane (i) passes through the point (1, 2, 3), we get

$3(1) + 4(2) - 5(3) + k = 0$

$3 + 8 - 15 + k = 0$

$-4 + k = 0$

$k = 4$

On putting $k = 4$, in Eq. (i), we get required equation

i.e., $3x + 4y - 5z + 4 = 0$

103. (C) Given equations of the plane
$x = 3z + 4$

$\Rightarrow \frac{x-4}{3} = \frac{z-0}{1}$ ... (i)

and $y = 2z - 3$

$\Rightarrow \frac{y+3}{2} = \frac{z-0}{1}$ ... (ii)

Therefore, the required equation of the line intersected by two planes (i) + (ii)

$\frac{x-4}{3} = \frac{y+3}{2} = \frac{z-0}{1}$

Hence, the direction ratio of the line (iii) is $(a_1, b_1, c_1) = (3, 2, 1)$

104. (B) The equation of the straight line passing through (a, b, c) and parallel to z-axis is

$\frac{x-a}{0} = \frac{y-b}{0} = \frac{z-c}{1}$

105. (B) Given, $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

$= \lim_{x \to 0} \frac{\sqrt{1+x}-1}{x} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$

[rationalization]

$= \frac{1+x-1}{x(\sqrt{1+x}+1)} = \lim_{x \to 0} \frac{1+x-1}{x(\sqrt{1+x}+1)}$

$= \lim_{x \to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$

106. (D) Given,

$\lim_{x \to 0} \frac{2(1-\cos x)}{x^2}$

$= \lim_{x \to 0} \frac{2\left(1-1+2\sin^2 \frac{x}{2}\right)}{x^2}$

$= \lim_{x \to 0} \frac{4\sin^2 \frac{x}{2}}{x^2} = \lim_{x \to 0} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2$

$= (1)^2 = 1$

$\left(\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right)$

107. (B) **Statement 1**

Given $\lim_{x \to 0} \frac{1}{x}$ exists.

LHL $= f(0-0) = \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} e^{-\frac{1}{h}} = -\infty$

RHL $= f(0+0) = \lim_{h \to 0} (0+h)$

$= \lim_{h \to 0} e^{\frac{1}{h}} = e^{\infty} = \infty$

$\because$ LHL $\neq$ RHL

$\therefore \lim_{x \to 0} e^{\frac{1}{x}}$ does not exist.

Hence, only statement 2 is true.

108. (C) Given, $x^m + y^m = 1$

Differentiating both sides *wrt* x, we get

$mx^{m-1} + my^{m-1} \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{mx^{m-1}}{my^{m-1}}$

$= \frac{-x^{m-1}}{y^{m-1}} = -\left(\frac{x^m}{y^m}\right)\left(\frac{y}{x}\right)$

$\Rightarrow$ Given, $\frac{-x}{y} = \left(\frac{x^m}{y^m}\right)\left(\frac{y}{x}\right)$

$\Rightarrow x^{m-2} = y^{m-2}$

Which is true when $m = 2$

109. (B) $f(x) = \frac{x^2}{|x|}, x \neq 0$

$f(0) = 0$

or $f(x) = \begin{cases} \frac{x^2}{x} = x & \text{if } x > 0 \\ \frac{x^2}{-x} = -x & \text{if } x < 0 \end{cases}$

$f(0) = 0$

Clearly, it is a moidulus function and modulus function is continuous everywhere.

110. (B) $\lim_{x \to 2} \frac{x-2}{x^2-4} = \lim_{x \to 2} \frac{x-2}{(x+2)(x-2)}$

$= \lim_{x \to 2} \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$

111. (D) Given, $\frac{dr}{dt} = 3$ cm/s

Since, area of circle (A) = πr^2

On differentiating it *wrt* time t we get

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi \times 10 \times 3$ $\quad [\because r = 10 \text{ cm}]$

$= 60\pi$ cm^2/s

112. (B) $f^{-1}(x) = \frac{x+5}{3}$

$\Rightarrow f^{-1}(y) = \frac{y+5}{3}$

$\because y = f(x)$

$\Rightarrow x = f^{-1}(y)$

$\Rightarrow x = \frac{y+5}{3}$

$\Rightarrow 3x = y + 5$

$\Rightarrow y = 3x - 5$

$\Rightarrow f(x) = 3x - 5$

113. (C) **Statement 1**

Given, $y = \text{In}(\sec x + \tan x)$

On differentiating it *wrt* x, we get

$\frac{dy}{dx} = \frac{1}{(\sec x + \tan x)} \frac{d}{dx}(\sec x + \tan x)$

$= \frac{1}{(\sec x + \tan x)} (\sec x.\tan x + \sec^2 x)$

$\Rightarrow = \frac{1}{(\sec x + \tan x)} \sec x (\tan x + \sec x)$

$= \sec x$

Statement 2

Given, $y = \log(\text{cosec } x - \cot x)$

$\frac{dy}{dx} = \frac{1}{(\text{cosec } x - \cot x)} \frac{d}{dx}(\text{cosec } x - \cot x)$

$= \frac{1}{(\text{cosec } x - \cot x)} (-\text{cosec } x.\cot x + \text{cosec}^2 x)$

$= \text{cosec } x.\frac{(\text{cosec } x - \cot x)}{(\text{cosec } x - \cot x)} = \text{cosec } x$

So, Statement 1 and 2 both are true.

114. (D) Given, $f(x) = 2^{\sin x}$

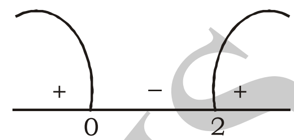
On differentiating it *wrt* x, we get

$\frac{d}{dx} f(x) = \frac{d}{dx}(2^{\sin x})$

$= (2)^{\sin x} \log 2.\frac{d}{dx}(\sin x)$

$= 2^{\sin x} \cos x \log 2$

115. (C)



Given, $f(x) = x^3 - 3x^2 + 6$

On differentating it *wrt* x, we get

$f'(x) = 3x^2 - 6x = 3x(x-2)$

For increasing,

$f'(x) > 0 \Rightarrow 3x(x-2) > 0$

$\therefore \quad x \in (-\infty, 0) \cup (2, \infty)$

$\Rightarrow \quad x > 2$ or $x < 0$

116. (A) **Statement 1**

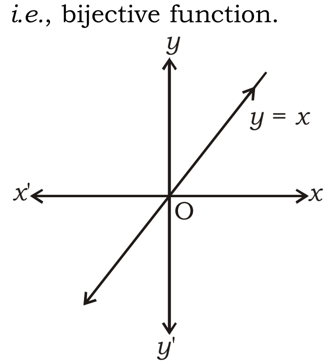
Given, $f(x) = x^3$ and $g(y) = y^3$.

Since, both the functions are identical.

$\therefore \quad f = g$

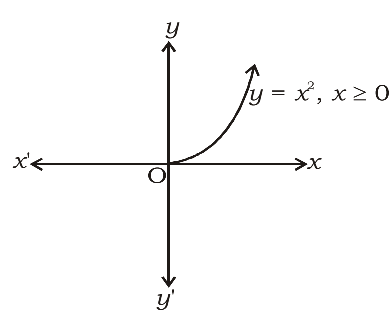
Statement 2

We know that, an identify function $f(x) = y = x$ is always one-one function, *i.e.*, bijective function.



Hence, only statement 1 is true.

117. (C)



Given, $A = \{x \in R \mid x > 0\}$

$f : A \to A, f(x) = x^2$

From adjoining figure $f(x)$ is one-one and onto, so its inverse exists.

Let $f(x) = y$

$\therefore x^2 = y$

$\Rightarrow x = \sqrt{y}$

$\Rightarrow f^{-1}(x) = \sqrt{x}$

So, f is not its own inverse.

118. (B) $y = \text{In}(e^{mx} + e^{-mx})$

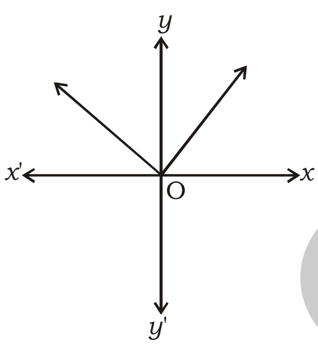
On differentiating it *wrt* x, we get

$$\frac{dy}{dx} = \frac{1}{e^{mx} + e^{-mx}}(me^{mx} - me^{-mx})$$

$$= \left[\frac{m(e^{mx} - e^{-mx})}{e^{mx} + e^{-mx}}\right]$$

$$\therefore \left(\frac{dy}{dx}\right)_{\text{at } x = 0} = m\left(\frac{1-1}{1+1}\right) = 0$$

119. (B)



Let $y = |x|$

$$y = \begin{cases} x, & x \ge 0 \\ -x & x < 0 \end{cases}$$

So, the minimum value of $|x|$ is zero.

120. (C) Let $I = \int a^x e^x \, dx$

$$= a^x \int e^x dx - \int\left[\left(\frac{d}{dx}a^x\right)\int e^x \, dx\right]dx$$

$$= a^x.e^x - \int a^x . \log a . e^x \, dx$$

$$I = a^x.e^x - \log a \int a^x.e^x \, dx$$

$$I = a^x.e^x - \log a.I$$

$$I (1 + \log a) = a^x e^x$$

$$I (\log e + \log a) = a^x e^x$$

$$I. \log ae = a^x e^x + C \Rightarrow I = \frac{a^x e^x}{\log(ae)} + C$$

NDA (MATHS) MOCK TEST - 59 (SOLUTION)

1. (A)	26. (C)	51. (C)	76. (A)	101. (A)
2. (C)	27. (C)	52. (C)	77. (C)	102. (C)
3. (B)	28. (C)	53. (C)	78. (B)	103. (C)
4. (B)	29. (C)	54. (A)	79. (B)	104. (B)
5. (B)	30. (C)	55. (B)	80. (B)	105. (B)
6. (D)	31. (D)	56. (D)	81. (B)	106. (D)
7. (C)	32. (B)	57. (B)	82. (C)	107. (B)
8. (B)	33. (C)	58. (B)	83. (D)	108. (C)
9. (D)	34. (A)	59. (C)	84. (C)	109. (B)
10. (A)	35. (C)	60. (D)	85. (B)	110. (B)
11. (A)	36. (A)	61. (A)	86. (B)	111. (D)
12. (D)	37. (D)	62. (B)	87. (A)	112. (B)
13. (B)	38. (B)	63. (D)	88. (B)	113. (C)
14. (A)	39. (B)	64. (C)	89. (B)	114. (D)
15. (B)	40. (B)	65. (B)	90. (C)	115. (C)
16. (B)	41. (A)	66. (D)	91. (C)	116. (A)
17. (B)	42. (B)	67. (D)	92. (D)	117. (C)
18. (A)	43. (B)	68. (D)	93. (D)	118. (B)
19. (B)	44. (C)	69. (B)	94. (B)	119. (B)
20. (B)	45. (C)	70. (A)	95. (C)	120. (C)
21. (A)	46. (B)	71. (C)	96. (B)	
22. (C)	47. (D)	72. (B)	97. (C)	
23. (A)	48. (B)	73. (D)	98. (A)	
24. (D)	49. (A)	74. (D)	99. (A)	
25. (D)	50. (A)	75. (D)	100. (A)	