NDA MATHS MOCK TEST - 70 (SOLUTION)

1. (B) A = $\{1,2,3\}$ and B = $\{2,3,4\}$

Then $(A \cup B) = \{1,2,3,4\}$

and $(A \cap B) = \{2,3\}$

$(A \cup B) \times (A \cap B) = \{1,2,3,4\} \times \{2,3\}$

$(A \cup B) \times (A \cap B)$

$= \{(1,2),(1,3)(2,2)(2,3),(3,2)(3,3),(4,2),(4,3)\}$

2. (B) $(E-(E-(E-(E-A))))$

$= (E-(E-(E-A')))$

$= (E-(E-A))$

$= (E-A')$

= A

$= B \cup C$

3. (C) Function g $(x) = 3x - 5$

Let g $(x) = y$

$y = 3x - 5$

$y + 5 = 3x$

$3x = y + 5$

$x = \dfrac{y+5}{3}$

$g^{-1}(x) = \dfrac{x+5}{3}$

4. (A) $\left(\dfrac{1-\sqrt{3}i}{2}\right)^{36} + \left(\dfrac{-1-\sqrt{3}i}{2}\right)^{36}$

we know that $\dfrac{-1+\sqrt{3}i}{2} = \omega$ and $\dfrac{-1-\sqrt{3}i}{2} = \omega_2$

$= (-\omega)^{36} + (\omega^2)^{36}$

$= (\omega^3)^{12} + \omega^{72}$

$= (\omega^3)^{12} + (\omega^3)^{24}$

$= (1)^{12} + (1)^{24}$ $[\because \omega^3 = 1]$

= 1 + 1

= 2

I statement is correct

II. $\sqrt{2i} - \sqrt{-2i}$

$= \sqrt{2i} - \sqrt{(-1)\times 2i}$

$= \sqrt{2i} - \sqrt{i^2 \times 2i}$ $[\because i^2 = -1]$

$= \sqrt{2}i(1-i)$

Modulus of $\sqrt{2}i(1-i) = \left|\sqrt{2}i(1-i)\right|$

$= \left|\sqrt{2}i\right| \left|(1-i)\right|$

$= \sqrt{2} \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{2} \times \sqrt{2}$

= 2

Modulus of $\sqrt{2}i(1-i) = 2$

II. Statement is incorrect

5. (D) Statement (S)

$\left(\dfrac{-1+\sqrt{-3}}{2}\right)^{30} + \left(\dfrac{-1-\sqrt{-3}}{2}\right)^{30}$

$\left(\dfrac{-1+\sqrt{3}i}{2}\right)^{30} + \left(\dfrac{-1-\sqrt{3}i}{2}\right)^{30}$ $\left[\because \sqrt{-1} = i\right]$

We know that

$\dfrac{-1+\sqrt{3}i}{2} = \omega$ $\dfrac{-1-\sqrt{3}i}{2} = \omega^2$

$w^{30} + (w^2)^{30}$

$= (w^3)^{10} + (w^2)^{30}$

$= (1)^{10} + (1)^{20}$ $\left[\because \omega^3 = 1\right]$

= 2

Statement (S) is false.

Reason (R) $\omega^3 = 1$

Reason (R) is true

6. (B) $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$

Let root of the equation = r, r

Sum of the roots $r + r = -\dfrac{[-2b(a+c)]}{a^2+b^2}$

$r = \dfrac{b(a+c)}{a^2+b^2}$

Product of the roots $r \times r = \dfrac{b^2+c^2}{a^2+b^2}$

$r^2 = \dfrac{b^2+c^2}{a^2+b^2}$

$r^2 = \frac{b^2+c^2}{a^2+b^2}$ and $r = \frac{b(a+c)}{a^2+b^2}$

putting the value of r

$$\left(\frac{b(a+c)}{a^2+b^2}\right)^2 = \frac{b^2+c^2}{a^2+b^2}$$

$$\frac{b^2(a+c)^2}{(a^2+b^2)^2} = \frac{(b^2+c^2)}{(a^2+b^2)}$$

$b^2(a+c)^2 = (b^2+c^2)(a^2+b^2)$

$b^2(a^2+c^2+2ac) = b^2a^2 + b^4 + c^2a^2 + b^2c^2$

$b^2a^2 + b^2c^2 + 2ab^2c = b^2a^2 + b^4 + c^2a^2 + b^2c^2$

$2ab^2c = b^4 + c^2a^2$

$b^2 + c^2a^2 - 2acb^2 = 0$

$(b^2 - ac)^2 = 0$

$b^2 - ac = 0$

$b^2 = ac$

7. (C) $n!$, $3\times(n!)$ and $(n+1)!$ are in G.P.

$[3\times n!]^2 = n! \times (n+1)!$

[$\because$ if a, b, c are in G.P. then $b^2 = ac$]

$9\times(n!)\times(n!) = (n!)\times(n+1)!$

$9\times n! = (n+1)\,n!$

$9 = n+1$

$n = 9-1$

$n = 8$

8. (D) Here terms of GP a, ar, ar^2

$a = \frac{1}{4}(ar + ar^2)$

$4a = ar + ar^2$

$4a = a(r + r^2)$

$r^2 + r - 4 = 0$

$$r = \frac{-1\pm\sqrt{(1)^2 - 4\times1(-4)}}{2\times1}$$

$$r = \frac{-1\pm\sqrt{1+16}}{2}$$

$$r = \frac{-1\pm\sqrt{17}}{2}$$

$$r = \frac{-1+\sqrt{17}}{2}, \frac{-1-\sqrt{17}}{2}$$

9. (A) Two dice are thrown.

n(s) = 6 × 6 = 36

E = [(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)]

[$\because$ sum is 8]

$n(E) = 5$

Probability = $\frac{n(E)}{n(S)}$

$= \frac{5}{36}$

10. (C) $x = 1101$

$1\times2^0 = 1$

$0\times2^1 = 0$

$1\times2^2 = 4$

$1\times2^3 = 8$

13

$y = 110$

$0\times2^0 = 0$

$1\times2^1 = 2$

$1\times2^2 = 4$

6

$x^2 + y^2 = (13)^2 + (6)^2$

$= 169 + 36$

$= 205$

$x^2 + y^2 = (205)_{10} = (11001101)_2$

2	205	
2	102	1
2	51	0
2	25	1
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

11. (B)

2	13	
2	6	1
2	3	0
2	1	1
	0	1

0.0625
×2
[0].1250
×2
[0].2500
×2
[0].5000
×2
[1].0000

$(13)_{10} = (1101)_2$

$(0.0625)_{10} = (10.0001)_2$

$(13.0625)_{10} = (10.0001)2$

$(13.0625)_{10} = (1101.0001)_2$

12. (C) **Statement** I:

2	325	
2	162	1
2	81	0
2	40	1
2	20	0
2	10	0
2	5	0
2	2	1
	1	0

$(325)_{10} = (101000101)_2$
Statements I is true
Statements II is true

13. (C) $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4\theta}}}$

$= \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 4\theta)}}}$

$= \sqrt{2+\sqrt{2+\sqrt{2.2\cos^2 2\theta}}}$

[$\because$ cos 2A = 2 cos² A–1, 1 + cos² A = 2cos² A]

$= \sqrt{2+\sqrt{2+2\cos 2\theta}}$

$= \sqrt{2+\sqrt{2(1+\cos 2\theta)}}$

$= \sqrt{2+\sqrt{2.2\cos^2\theta}}$

$= \sqrt{2+2\cos\theta}$

$= \sqrt{2(1+\cos\theta)}$

$= \sqrt{2.2\cos^2\frac{\theta}{2}}$

$= 2\cos\frac{\theta}{2}$

14. (B) sin20° sin40° sin60° sin80°

$= \frac{1}{2}$ sin20° sin60° (2sin40°. sin80°)

$= \frac{1}{2}\sin 20° \times \frac{\sqrt{3}}{2}$ (2 sin80°. sin40°)

$= \frac{\sqrt{3}}{4}\sin 20° [\cos(80°-40°) - \cos(80°+40°)]$

[$\because$ 2sinA. sinB = cos (A–B) – cos (A + B]

$= \frac{\sqrt{3}}{4}\sin 20° (\cos 40° - \cos 120°)$

$= \frac{\sqrt{3}}{4}\sin 20° [1 - 2\sin^2 20° + \sin 30°]$

[$\because$ cos 2A = 1 – 2sin²A]

$= \frac{\sqrt{3}}{4}\sin 20° [1 - 2\sin^2 20° + \frac{1}{2}]$

$= \frac{\sqrt{3}}{4}\sin 20° [3 - 4\sin^2 20°]$

$= \frac{\sqrt{3}}{4} [3\sin 20° - 4\sin^3 20°]$

[$\because$ sin 3A = 3 sinA – 4 sin³A]

$= \frac{\sqrt{3}}{4} \times \sin(3\times 20) \Rightarrow \sin 60 = \frac{\sqrt{3}}{4}\times\frac{\sqrt{3}}{4}$

$= \frac{3}{16}$

15. (B) In DABC, LA, LB, LC are in A.P.
Let angles B –P B B + P
B – P + B + B + P = 180
3B = 180
B = 60

Angle ∠A = 60 – P ∠B = 60 ∠B = 60+P

Given that b : c = $\sqrt{3} : \sqrt{2}$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{Sin\ C}{Sin\ B} = \frac{c}{b}$

$\frac{Sin\ C}{Sin\ 60} = \frac{\sqrt{2}}{\sqrt{3}}$

$\frac{(Sin\ C)\times 2}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$

Sin C = $\frac{1}{\sqrt{2}}$

Sin C = sin 45°
C = 45°

16. (C)

17. (B) B = 60 C = 45 then A = 75

$\sin 75 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 75} = \frac{b}{\sin 60} = \frac{c}{\sin 45}$$

$$\frac{a \times 2\sqrt{2}}{\left(\sqrt{3}+1\right)} = \frac{b+2}{\sqrt{3}} = \frac{c+\sqrt{2}}{1}$$

$$\frac{a \times 2\sqrt{2}}{\left(\sqrt{3}+1\right) \times 2\sqrt{2}} = \quad = \frac{c \times \sqrt{2}}{2 \times \sqrt{2}}$$

$$\frac{a}{\sqrt{3}+1} = \frac{b}{\sqrt{6}} = \frac{c}{2}$$

a : b : c = $\sqrt{3}+1 : \sqrt{6} : 2$

$$= \frac{\sqrt{3}+1}{\sqrt{2}} : \frac{\sqrt{6}}{\sqrt{2}} : \frac{2}{\sqrt{2}}$$

a : b : c = $\frac{\sqrt{3}+1}{\sqrt{2}} : \sqrt{3} : \sqrt{2}$

18. (C) A scalene triangle

19. (C) $\text{Sin}^2A + \text{Sin}^2B + \text{Sin}^2C$

$= \text{Sin}^2 75 + \text{Sin}^2 60 + \text{Sin}^2 45$

$$= \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{3+1+2\sqrt{3}}{4 \times 2} + \frac{3}{4} + \frac{1}{2}$$

$$= \frac{2\left(2+\sqrt{3}\right)}{4 \times 2} + \frac{3}{4} + \frac{1}{2}$$

$$= \frac{7+\sqrt{3}}{4}$$

20. (C) Ratio of triangle 1 : 2 : 7

$x, 2x, 7x$

$x + 2x + 7x = 180$

$10x = 180$

$x = 18$

and 18, 36, 126

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{greatest\ side}{\sin allest\ side}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{gretest\ side}{smallest\ side} \quad \frac{c}{a} = \frac{\sin C}{\sin A}$$

$$\frac{c}{a} = \frac{\sin 126}{\sin 18}$$

$$= \frac{\cos 36}{\sin 18}$$

$$= \frac{\frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}-1}{4}}$$

$$\frac{c}{a} = \frac{\sqrt{5}+1}{\sqrt{5}-1}$$

21. (D) Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = (k)$$

a = K sin A b = K sin B c = K sin C

$$\frac{a+c}{b} = \frac{K\sin A + K\sin C}{K\sin B}$$

$$= \frac{K\left(\sin A + \sin C\right)}{K\sin B}$$

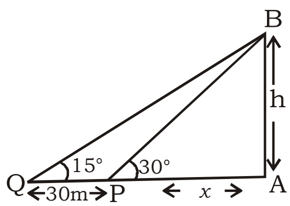
$$= \frac{2\sin\frac{A+C}{2}.\cos\frac{A-C}{2}}{2\sin\frac{B}{2}.\cos\frac{B}{2}}$$

$$= \frac{\sin\left(\frac{180-B}{2}\right)\cos\frac{1}{2}(A-C)}{\sin\frac{B}{2}.\cos\frac{B}{2}}$$

$$= \frac{\cos\frac{B}{2}\cos\frac{A-C}{2}}{\sin\frac{B}{2}.\cos\frac{B}{2}}$$

$$\frac{a+c}{b} = \frac{\cos\frac{A-C}{2}}{\sin\frac{B}{2}}$$

22. (B) Let height of tower = h
and AP = x



In ΔABP

$$\tan 30 = \frac{h}{x} = \frac{AB}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

x = $\sqrt{3}h$

In ΔABQ

$$\tan(15) = \frac{AB}{AD}$$

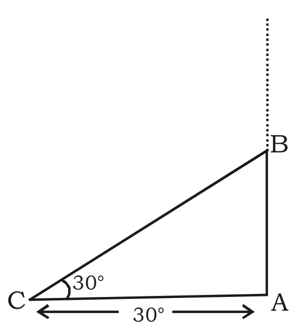
$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h}{30+x}$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h}{30+\sqrt{3}h}$$

$$\left(\sqrt{3}-1\right)\left(30+\sqrt{3}h\right) = \left(\sqrt{3}+1\right)h$$

On solving
h = 15
height of tower is 15m

23. (B) Broken part of the tree = BC



In ΔABC

$$\cos 30 = \frac{AC}{BC}$$

$$\frac{\sqrt{3}}{2} = \frac{30}{BC}$$

$$BC = \frac{30\times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$BC = 20\sqrt{3}$$

24. (D) $Cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$

$$\text{Cot}^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}} \times \frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$$

$$\text{Cot}^{-1}\left[\frac{\left(\sqrt{1-\sin x}+\sqrt{1+\sin x}\right)^2}{\left(\sqrt{1-\sin x}\right)^2-\left(\sqrt{1+\sin x}\right)^2}\right]$$

$$\text{Cot}^{-1}\left[\frac{2+2\sqrt{1-\sin^2 x}}{-2\sin x}\right]$$

$$\text{Cot}^{-1}\left[\frac{2+2\sqrt{\cos^2 x}}{-2\sin x}\right]$$

$$\text{Cot}^{-1}\left[\frac{2+2\cos x}{-2\sin x}\right]$$

$$\text{Cot}^{-1}\left[\frac{2(1+\cos x)}{-2\sin x}\right]$$

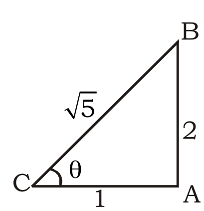
$$= \text{Cot}^{-1}\left[\frac{2\times\cos^2\frac{x}{2}}{-2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$$

$$= \text{Cot}^{-1}\left[-\cot\frac{x}{2}\right]$$

$$= \text{Cot}^{-1}\left[\cot\left(\pi-\frac{x}{2}\right)\right]$$

$$= \pi - \frac{x}{2}$$

25. (B) $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = \theta$



$\cos q = \frac{1}{\sqrt{5}}$

$\sec q = \sqrt{5}$

$q = \sec^{-1}\left(\sqrt{5}\right)$

$\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$

$\operatorname{cosec}^{-1}\left(\sqrt{5}\right) + \sec^{-1}\left(\sqrt{5}\right) = \frac{\pi}{2}$

$\operatorname{cosec}^{-1}\left(\sqrt{5}\right) = \frac{\pi}{2} - \sec^{-1}\left(\sqrt{5}\right)$

$\operatorname{cosec}^{-1}\left(\sqrt{5}\right) = \frac{\pi}{2} - q$

26. (C) $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$

$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = \sin\frac{\pi}{2}$

on comparing

$\sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$

$\sin^{-1}\frac{1}{5} = \frac{\pi}{2} - \cos^{-1}x$

$\sin^{-1}\frac{1}{5} = \sin^{-1}x$

$x = \frac{1}{5}$

27. (C) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}$

$\sin^{-1}x = \frac{\pi}{2}$, $\sin^{-1}y = \frac{\pi}{2}$, $\sin^{-1}z = \frac{\pi}{2}$

$x = \sin\frac{\pi}{2}$, $y = \sin\frac{\pi}{2}$, $z = \sin\frac{\pi}{2}$

$x = 1$, $y = 1$, $z = 1$

Value of $x^{2001} + y^{2002} + z^{2003}$

(1)2001 + (1)2002 + (1)2003

1 + 1 + 1 = 3

28. (C) $x^4 + y^4 + z^4 - 4x^2y^2 - 4y^2z^2 + 4x^2z^2 - 3xy^2z$

$= (1)^4 + (1)^4 + (1)^4 - 4(1)^2(4)^2 - 4(1)^2(1)^2 + 4(1)^2(1)^2 - 3 \times 1 \times (1)^2 - 3 \times 1 \times (x)^2 \times 1$

$= 1 + 1 + 1 - 4 - 4 + 4 - 3$

$= 3 - 4 - 3$

$= -4$

29. (B) $\sin(\sin^{-1}x + \sin^{-1}y) = \sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$

$= \sin p$

$= 0$

30. (D) $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right]$

$\tan\left[\tan^{-1}\left\{\frac{2 \times \frac{1}{5}}{1 - \left(\frac{1}{5}\right)^2}\right\} - \frac{\pi}{4}\right]$

$\left[\because \tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$\tan\left[\tan^{-1}\left(\frac{\frac{2}{5}}{1 - \frac{1}{25}}\right) - \frac{\pi}{4}\right]$

$\tan\left[\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}(1)\right]$

$\tan\left[\tan^{-1}\left(\frac{\frac{5}{12} - 1}{1 + \frac{5}{12} \times 1}\right)\right]$

$\left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$\tan\left[\tan^{-1}\left(\frac{\frac{-7}{12}}{\frac{17}{12}}\right)\right]$

$\tan\left[\tan^{-1}\left(\frac{-7}{17}\right)\right] = \frac{-7}{17}$

31.(D) $\sin\left[\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{3}{4}\right]$

$= \sin\left[\tan^{-1}\left(\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\times\frac{3}{4}}\right)\right]$

$= \sin\left[\tan^{-1}\left(\frac{56}{33}\right)\right]$

$= \sin\left[\sin^{-1}\left(\frac{56}{65}\right)\right]$

$= \frac{56}{65}$

32. (D) $\tan\left[2\tan^{-1}\frac{3}{7}+\sin^{-1}\frac{3}{5}\right]$

$\tan\left[\tan^{-1}\left(\frac{2\times\frac{3}{7}}{1-\left(\frac{3}{7}\right)^2}\right)+\tan^{-1}\frac{\frac{3}{5}}{\sqrt{\left(1-\left(\frac{3}{5}\right)^2\right)}}\right]$

$\left[\because \sin^{-1}x=\tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$

$\tan\left[\tan^{-1}\left(\frac{\frac{6}{7}}{1-\frac{9}{49}}\right)+\tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\frac{9}{25}}}\right)\right]$

$\tan\left[\tan^{-1}\left(\frac{21}{20}\right)+\tan{-1}\left(\frac{\frac{3}{5}}{\frac{4}{5}}\right)\right]$

$\tan\left[\tan^{-1}\left(\frac{21}{20}\right)+\tan^{-1}\frac{3}{4}\right]$

$\tan\left[\tan^{-1}\left(\frac{\frac{21}{20}+\frac{3}{4}}{1-\frac{21}{20}\times\frac{3}{4}}\right)\right]$

$\tan\left[\tan^{-1}\left(\frac{36}{17}\right)\right] = \frac{36}{17}$

33. (D) $t(x) = \sqrt{1-\frac{1}{x}}$

$\tan(x) = \sqrt{\frac{x-1}{x}}$

$x \neq 0$ $\quad \frac{x-1}{x} > 0$

$x < 0, \quad x > 1$

domain of $t(x) = (-\infty, 0) \cup (1, \infty)$

34. (D) $t(x) = ax + b$ and $g(x) = cx + d$

$tog(x) = got(x)$

$t[g(x)] = g[t(x)]$

$t[(cx + d)] = g(ax + b)$

$a(cx + d) + b = c(ax + b) + d$

$ad + b = bc + d$

$ad + b = cb + d$

$t(d) = g(b)$

35. (B) $\lim_{x\to a}\frac{\log(x-a)}{\log(e^x-e^a)}$ $\quad\left(\frac{\infty}{\infty}\right)$ form

by L - Hospital's Rule

$= \lim_{x\to a}\frac{\frac{d}{dx}\log(x-a)}{\frac{d}{dx}\log(e^x-e^a)}$

$= \lim_{x\to a}\frac{e^x-e^a}{e^x(x-a)}$ $\quad\left[\frac{0}{0}\right]$form

by L-Hospital is Rule

$= \lim_{x\to a}\frac{\frac{d}{dx}(e^x-e^a)}{\frac{d}{dx}\{e^x(x-a)\}}$

$= \lim_{x\to a}\frac{e^x-0}{e^x.1-(x-a)e^x}$

$= \lim_{x\to a}\frac{e^x}{e^a-xe^x+ae^x}$

$= \frac{e^a}{e^a-ae^a+ae^a} = \frac{e^a}{e^a} = 1$

36. (A) $\lim_{x\to\infty}\left\{x\sin\frac{2}{x}\right\}$

$= \lim_{x\to\infty}\left\{\frac{\sin\frac{2}{x}}{\frac{1}{x}}\right\}$

$= \lim_{x\to\infty}\left\{\frac{2\sin\frac{2}{x}}{\frac{2}{x}}\right\}$

$= \lim_{x\to\infty}\left\{2\left(\frac{\sin\frac{2}{x}}{\frac{2}{x}}\right)\right\}$

$= \lim_{x\to\infty} 2 \lim_{x\to\infty}\left(\frac{\sin\frac{2}{x}}{\frac{2}{x}}\right)$

$= 2 \times 1$

$= 2$

37. (B) $\lim_{x\to0}\left[\frac{1+\tan x}{1+\sin x}\right]^{\operatorname{cosec} x}$

$= \lim_{x\to0}\frac{\left[1+\frac{\sin x}{\cos x}\right]^{\frac{1}{\sin x}}}{\left[1+\sin x\right]^{\frac{1}{\sin x}}}$

$= \lim_{x\to0}\frac{\left[\left(1+\frac{\sin x}{\cos x}\right)^{\frac{\cos x}{\sin x}}\right]^{\frac{1}{\cos x}}}{\left[1+\sin x\right]^{\frac{1}{\sin x}}}$

$= \frac{e^{\lim_{x\to0}\frac{1}{\cos x}}}{e}$ $\quad\left[\lim_{x\to a}\left[\{1+t(x)\}^{\frac{1}{t(x)}} = e\right]\right]$

$= \frac{e^{\frac{1}{\cos 0}}}{e}$

$= \frac{e}{e} = 1$

38. (D) $t(x) = |x-5|$

$\lim_{x\to5^+} t(x) = \lim_{h\to0} t(5+h)$

$= \lim_{h\to0} |5+h-5|$

$= \lim_{h\to0} |h|$

$\lim_{x\to5^+} t(x) = 0$

$\lim_{x\to5^-} t(x) = \lim_{h\to0} t(5-h)$

$= \lim_{h\to0} |5-h-5|$

$= \lim_{h\to0} |-h|$

$= \lim_{h\to0} h$

$\lim_{x\to5^-} t(x) = 0$

$t(x)$ is continuous at $x = 5$

39.(B) $t(x)\begin{cases}4x+5 & 0\le x\le 2\\ 3x+\lambda & 2<x<3\end{cases}$

$\lim_{x\to2^+} t(x) = \lim_{x\to2^-} t(x)$

$\lim_{x\to2}(3x+\lambda) = \lim_{x\to2}(4x+5)$

$3\times2+\lambda = 4\times2+5$

$6+\lambda = 8+5$

$6+\lambda = 13$

$\lambda = 13-6$

$\lambda = 7$

40. (B) $t(x) = \begin{cases}\frac{1-\sin x}{\pi-2x} & x\ne\frac{\pi}{2}\\ \lambda & x=\frac{\pi}{2}\end{cases}$

$\lim_{x\to\frac{\pi}{2}^+} t(x) = \lim_{x\to\frac{\pi}{2}^-} t(x) = t\left(\frac{\pi}{2}\right)$

$\lim_{x\to\frac{\pi}{2}^+} t(x) = t\left(\frac{\pi}{2}\right)$

$\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\pi-2x} = 1$

by L-Hospital's Rule

$\lim_{x\to\frac{\pi}{2}}\frac{\frac{d}{dx}(1-\sin x)}{\frac{d}{dx}(\pi-2x)} = 1$

$\lim_{x\to\frac{\pi}{2}}\frac{0-\cos x}{0-2} = 1$

$\frac{\cos\frac{\pi}{2}}{+2} = 1$

$\frac{0}{2} = 1$

$1 = 0$

41. (B) $\lim_{x\to\infty}\left(1-\frac{4}{x-1}\right)^{3x-1}$

$\lim_{x\to\infty}\left\{\left[1+\left(\frac{-4}{x-1}\right)\right]^{-\frac{(x-1)}{4}}\right\}^{(3x-1)\times\left(\frac{-4}{x-1}\right)}$

$= e^{\lim_{x\to\infty}-4\left(\frac{3x-1}{x-1}\right)}$

$= e^{\lim_{x\to\infty}-4\left(\frac{3-\frac{1}{x}}{1-\frac{1}{2}}\right)}$

$= e^{-4\left(\frac{3-0}{1-0}\right)}$

$= e^{-4\times 3}$

$= e^{-12}$

42. (D) $t(x) = \begin{cases} 4\times 5^x & x<0 \\ 8a+x & x\ge 0\end{cases}$ is continuous at $x = 0$

then $\lim_{x\to 0^+} t(x) = \lim_{x\to 0^-} t(x)$

$\lim_{x\to 0^+} 8a + x = \lim_{x\to 0^-} 4\times 5^x$

$8a + 0 = 4 \times 5^0$

$8a = 4 \times 1$

$a = \frac{1}{2}$

43. (B) $y = e^{\sin^2\left(\sqrt{x^2-1}\right)}$

$\frac{dy}{dx} = \frac{d}{dx} e^{\sin^2\left(\sqrt{x^21}\right)}$

$= e^{\sin^2\left(\sqrt{x^2-1}\right)} \frac{1}{dx}\left\{\sin^2\sqrt{x^2-1}\right\}$

$= e^{\sin^2\sqrt{x^2-1}}\ 2\sin\sqrt{x^2-1}\ \cos\sqrt{x^2-1}\ \frac{d}{dx}$

$(x^2-1)^{\frac{1}{2}}$

$= \sin(2\sqrt{x^2-1})\ e^{\sin^2\sqrt{x^2-1}}$

$\frac{1}{2(x^2-1)^{\frac{1}{2}}}\ \frac{d}{dx}(x^2-1)$

$= \frac{\sin 2\left(\sqrt{x^2-1}\right)}{2\sqrt{x^2-1}}\ e^{\sin^2\left(\sqrt{x^2-1}\right)}\ (2x-0)$

$= \frac{2x\sin 2\sqrt{x^2-1}}{2\sqrt{x^2-1}}\ e^{\sin^2\left(\sqrt{x^2-1}\right)}$

$\frac{dy}{dx} = \frac{x\sin 2\sqrt{x^2-1}}{\sqrt{x^2-1}}\ e^{\sin^2\left(\sqrt{x^2-1}\right)}$

44. (D) $y = e^{\cos x}$

$\frac{dy}{dx} = \frac{d}{dx}(e^{\cos x})$

$= e^{\cos x}\frac{d}{dx}(\cos x)$

$= e^{\cos x}(-\sin x)$

$\frac{dy}{dx} = -\sin x\ e^{\cos x}$

45. (C) $y = \log(1 + x^2)$ and $z = x^2 + 1$

$y = \log z$

$\frac{dy}{dx} = \frac{1}{z}$

46. (B) $x^m y^n = 2(x+y)^{m+n}$

taking log both side

$m\log x + n\log y = \log 2 + (m+n)\log(x+y)$

differentiate both w.z.t. 'x'.

$m\cdot\frac{1}{x} + n\frac{1}{y}\frac{dy}{dx} = 0 + (m+n)\frac{1}{x+y}\frac{d}{dx}(x+y)$

$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{(m+n)}{x+y}\left(1+\frac{dy}{dx}\right)$

$\frac{n}{y}\frac{dy}{dx} - \frac{m+n}{x+y}\frac{dy}{dx} = \frac{m+n}{x+y} - \frac{m}{x}$

$\frac{dy}{dx}\left(\frac{nx-my}{y}\right) = \left(\frac{nx-my}{x}\right)$

$\frac{dy}{dx} = \frac{y}{x}$

47. (D) $y = \frac{3at^2}{1+t^3}$

$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{3at^2}{1+t^3}\right)$

$= \frac{(1+t^3)\frac{d}{dt}(3at^2) - 3at^2\frac{d}{dt}(1+t^3)}{(1+t^3)^2}$

$= \frac{(1+t^3)\times 3a\times 2t - 3at^2\times(0+3t^2)}{(1+t^3)^2}$

$$\frac{dy}{dt} = \frac{6at(1+t^3)-9at^4}{(1+t^3)^2}$$

$$= \frac{6at+6at^4-9at^4}{(1+t^3)^2}$$

$$\frac{dy}{dt} = \frac{6at-3at^4}{(1+t^3)^2}$$

$$\frac{dy}{dt} = \frac{3at(2-t^3)}{(1+t^3)^2}$$

48. (C) $x = \frac{3at}{1+t^3}$

Differentiate both side *w.r.t.* '*t*'.

$$\frac{dx}{dt} = \frac{(1+t^3)\frac{d}{dt}(3at)-3at\frac{d}{dt}(1+t^3)}{(1+t^3)^2}$$

$$\frac{dx}{dt} = \frac{(1+t^3)\times 3a-3at(0+3t^2)}{(1+t^3)^2}$$

$$\frac{dx}{dt} = \frac{3a+3at^3-9at^3}{(1+t^3)^2}$$

$$\frac{dx}{dt} = \frac{3a-6at^3}{(1+t^3)^2}$$

$$\frac{dx}{dt} = \frac{3a(1-2t^3)}{(1+t^3)^2}$$

49. (B) We know that

$$\frac{dy}{dt} = \frac{3at(2-t^3)}{(1+t^3)^2} \text{ and } \frac{dx}{dt} = \frac{3a(1-2t^3)}{(1+t^3)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$$

$$= \frac{3at(2-t^3)}{(1+t^3)^2}\times\frac{(1+t^3)^2}{3a(1-2t^3)}$$

$$\frac{dy}{dx} = \frac{t(2-t^3)}{(1-2t^3)}$$

at $x = 1$

$$\left(\frac{dy}{dx}\right)_{x=1} = \frac{1(2-1^3)}{(1-2\times 1^3)}$$

$$= \frac{(2-1)}{(1-2)}$$

$$\left(\frac{dy}{dx}\right)_{dx=1} = \frac{1}{-1} = -1$$

50. (A) $\frac{dy}{dx} = \frac{t(2-t^3)}{(1-2t^3)}$

at $x = \frac{1}{2}$

$$\left(\frac{dy}{dx}\right)_{x=\frac{1}{2}} = \frac{\frac{1}{2}\left[2-\left(\frac{1}{2}\right)^3\right]}{\left[1-2\left(\frac{1}{2}\right)^3\right]}$$

$$= \frac{\frac{1}{2}\left[2-\frac{1}{8}\right]}{\left[1-2\times\frac{1}{8}\right]}$$

$$= \frac{a^2\times 2\cos 2\theta}{a\cos\theta}$$

$$\left(\frac{dy}{dx}\right)_{at\,x=\frac{1}{2}} = \frac{5}{4}$$

51. (B) Let $y = e^{x^3}$ and $z = \log x$

differentiate w.r.t. '*x*'.

$$\frac{dy}{dx} = \frac{d}{dx}(e^{x^3}) \text{ and } \frac{dz}{dx} = \frac{d}{dx}(\log x)$$

$$\frac{dy}{dx} = e^{x^3}\frac{d}{dx}(x^3) \text{ and } \frac{dz}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = e^{x^3}.3x^2 \quad \frac{dx}{dz} = x$$

differential *y w.r.t.* '*z*'

$$\frac{dy}{dz} = \frac{dy}{dx}\times\frac{dx}{dz}$$

$$= e^{x^3}\cdot 3x^2\times x$$

$$\frac{dy}{dz} = 3x^3 e^{x^3}$$

52. (D) $x = t^3$ and $y = 3t^4$

differentiate *w.r.t.* '*t*'

$$\frac{dx}{dt} = 3t^2 \text{ and } \frac{dy}{dt} = 3\times 4t^3$$

$$\frac{dt}{dx} = \frac{1}{3t^2} \text{ and } \frac{dy}{dt} = 12t^3$$

$$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$$

$= 12t^3 \times \frac{1}{3t^2}$

$\frac{dy}{dx} = 4t$

differentiate both side w.r.t. 'x'

$\frac{d^2y}{dx^2} = \frac{d}{dx}(4t)$

$\frac{d^2y}{dx^2} = 4\frac{dt}{dx}$

$\frac{d^2y}{dx^2} = 4 \times \frac{1}{3t^2}$

$\frac{d^2y}{dx^2} = \frac{4}{3t^2}$

53. (B) $y = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)$

Let $x = a$ sinq

$\frac{dx}{d\theta} = a$ cosq

$y = a\text{sinq}\sqrt{a^2 - a^2\sin^2\theta} + a^2\sin^{-1}\left(\frac{a\sin\theta}{a}\right)$

$y = a\text{sinq}.a\sqrt{(1-\sin^2\theta)} + a^2.\text{q}$

$y = \frac{2a^2}{2}\text{sinq.cosq} + a^2\cdot\text{q}$

$y = \frac{1}{2}a^2\ \text{sin2q} + a^2\text{q}$

Differentiate w.r.t 'x' ·

$\frac{dy}{dx} = \frac{1}{2}a^2\ \text{cos2q}\ (2)\frac{d\theta}{dx} + a^2.\frac{d\theta}{dx}$

$^2\ \frac{dy}{dx} = a^2 \times \frac{1}{a\cos\theta}(\cos 2\theta + 1)$

$\frac{dy}{dx} = \frac{a^2 \times 2\cos 2\theta}{a\cos\theta}$

$\frac{dy}{dx} = 2a\cos\theta$

$= 2a\sqrt{1-\sin^2\theta}$

$= 2a\sqrt{1-\left(\frac{x}{a}\right)^2}$

$= 2a\sqrt{\frac{a^2-x^2}{a^2}}$

$\frac{dy}{dx} = 2\sqrt{a^2-x^2}$

54. (A) $t(x) = 2x^3 - 15x^2 + 36x + 4$

differentiate both side w.r.t. 'x'

$t'(x) = 2 \times 3x^2 - 15 \times 2x + 36 \times 1 + 0$

$t'(x) = 6x^2 - 30x + 36$

$t'(x) = 6 \times 2x - 30 \times 1 + 0$

$t'(x) = 12x - 30$ ...(i)

for minimum and maximum

$t'(x) = 0$

$6x^2 - 30x + 36 = 0$

$6(x^2 - 5x + 6) = 0$

$x(x-2) - 3(x-2) = 0$

$(x-2)(x-3) = 0$

$x = 2, 3$

at $x = 2$

$t''(x) = 12x - 30$

$t''(2) = 24 - 30$

$= -6$(maxima)

$t(x)$ is maximum at $x = 2$

at $x = 3$

$t''(3) = 12 \times 3 - 30$

$= 36 - 20$

$= 6$ (minima)

55. (B) Projection of $\hat{a}$ on $\hat{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$= \frac{(I - 2j + k)\cdot(4i - 4j + \hat{k})}{\sqrt{(4)^2 + (-4)^2 + (7)^2 k}}$

$= \frac{4+8+7}{\sqrt{16+16+49}} = \frac{19}{\sqrt{81}} = \frac{19}{9}$

56. (C) $I = \int \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$

$I = \int \cos\left[\cos(-x)\right]dx$

$I = \int -x\ dx$

$I = -\frac{x^2}{2} + C$

$I = -\frac{1}{2}x^2 + C$

57. (B) $I = \int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

Let $\sqrt{x} = t$

differentiate both side

$\frac{1}{2}x^{-\frac{1}{2}}\,dx = at$

$\frac{dx}{\sqrt{x}} = 2at$

$I = \int \sin t\ (2at)$

$I = 2\int \sin t\ \ at$

$I = -2\cos t + c$

$I = -2\cos\sqrt{x} + c$

58. (C) $I = \int e^x\left(\sin^{-1}x + \frac{1}{\sqrt{1-x^2}}\right)dx$

$I = e^x \sin^{-1}x + C$

$\left[\because \int e^x\left(t(x)+t'(x)\right)dx = e^x\, t(x) + c\right]$

59. (D) $I = \int \frac{1}{2}e^{x^3}\left(6x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\right)dx$

$I = \int \frac{e^{x^3}(3x^2)}{3x^2 \times 2}\left(6x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\right)dx$

$I = \int e^{x^3}(3x^2)\left(\frac{6x^{\frac{3}{2}} - 3x^{\frac{1}{2}}}{6x^2}\right)dx$

$I = \int e^{x^3}(3x^2)\left[x^{\frac{-1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}\right]dx$

$I = e^{x^3}\cdot x^{\frac{-1}{2}} + c$

$\left[\because \int e^x\left(t(x) + t'(x)\right)dx = e^x.t(x) + c\right]$

$I = \frac{e^{x^3}}{\sqrt{x}} + c$

60. (B) $I = \int \frac{(x+1)^2}{x(x^2+1)}dx$

$I = \int \frac{x^2+1+2x}{x(x^2+1)}dx$

$I = \int \frac{(x^2+1)}{x(x^2+1)}dx + \int \frac{2x}{x(x^2+1)}dx$

$I = \int \frac{1}{x}dx + 2\int \frac{1}{1+x^2}dx$

$I = \log_e x + 2\tan^{-1}x + c$

61. (D) $I = \int \frac{e^x}{(2++e^x)(e^x+1)}dx$

$e^x = t$

$e^x\,dx = dt$

$I = \int \frac{dt}{(2+t)(t+1)}$

$I = \int\left(-\frac{1}{t+2} + \frac{1}{t+1}\right)dt$

$I = -\log(t+2) + \log(t+1) + c$

$I = \log\sqrt{(a-2)^2 + \left(b-2\sqrt{3}\right)^2} + c$

$I = \log\left(\frac{e^x+1}{e^x+2}\right) + c$

62. (A) $I = \int_0^1 x(1-x)^5\,dx$

Property IV $\int_0^a t(x)\,dx = \int_0^a t(a-x)\,dx$

$I = \int_0^1 (1-x)\left[1-(1-x)\right]^5 dx$

$I = \int_0^1 (1-x)x^5\,dx$

$I = \int_0^1 \left(x^5 - x^6\right)dx$

$I = \left[\frac{x^6}{6} - \frac{x^7}{7}\right]_0^1$

$I = \frac{1}{6} - \frac{1}{7}$

$= \frac{1}{42}$

63. (A) $I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1+\sin x\cdot\cos x}\,dx$ ...(i)

Property IV

$\int_0^a t(x)\,dx = \int_0^a t(a-x)\,dx$

$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-x\right) - \sin\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cdot\cos\left(\frac{\pi}{2}-x\right)}\,dx$

$I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\cos x\cdot\sin x}\,dx$ ...(ii) by adding equation (i) and equation (ii)

$2I = \int_0^{\frac{\pi}{2}} 0 dx$

$2I = 0$

$I = 0$

64. (C) $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ ...(i)

Property IV $\int_0^a t(x) = \int_0^a t(a-x)\,dx$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\, dx$$

$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x+\sin x}}\, dx$ ...(ii)

by adding equation (i) and equation (ii)

$2I = \int_0^{\frac{\pi}{2}} 1.dx$

$2I = [x]_0^{\frac{\pi}{2}}$

$2I = \frac{\pi}{2} - 0$

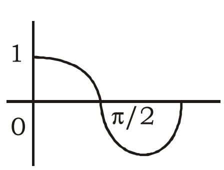
$I = \frac{\pi}{4}$

65. (D) $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos x|\, dx$

We know that

$$\int_{-a}^{a} t(x)\,dx = \begin{cases} 2\int_0^a t(x)\,dx & t(x) \text{ is even} \\ 0 & t(x) \text{ is odd} \end{cases}$$

$I = 2\int_0^{\frac{\pi}{2}} |\cos x|\, dx$



$I = 2\int_0^{\frac{\pi}{2}} \cos x\ dx$

$I = 2[\sin x]_0^{\frac{\pi}{2}}$

$I = 2\left[\sin\frac{\pi}{2} - \sin 0\right]$

$I = 2 \times 1$

$I = 2$

66. (B) $I = \int_0^{2\pi} [|\cos x| + |\sin x|]\ dx$

$I = \int_0^{2\pi} |\cos x|\, dx + \int_0^{2\pi} |\sin x|\, dx$

$I = \int_0^{\frac{\pi}{2}} \cos x\ dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos x)\,dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x\,dx$

$+ \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} (-\sin x)\,dx$

$I = [\sin x]_0^{\frac{\pi}{2}} - [\sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + [\sin x]_{\frac{3\pi}{2}}^{2\pi} + [-\cos x]_0^{\pi}$

$+ [\cos x]_{\pi}^{2\pi}$

$I = \left[\sin\frac{\pi}{2} - \sin 0\right] - \left[\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right] +$

$\left[\sin 2\pi - \sin\frac{3\pi}{2}\right] - [\cos\pi - \cos 0] + [\cos 2\pi$

$- \cos$p]

$I = [1 - 0] - [-1-1] + [0+1] - [-1 - 1] + [0]$

$I = 1 + 2 + 3$

$I = 6$

67. (C) $I = \int_0^1 \log\sin\left(\frac{\pi}{2}x\right)dx$

Let $\frac{\pi}{2}x = t$ Where $x \to 0,\ t \to 0$

$\frac{\pi}{2}dx = dt$ $x \to 1,\ t \to \frac{\pi}{2}$

$dx = \frac{2}{\pi}dt$

$I = \int_0^{\frac{\pi}{2}} \log\ \sin\ t\left(\frac{2}{\pi}dt\right)$

$I = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \log\ \sin\ t\ dt$

$I = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \log\sin x\,dx$

$\left[\because \int_0^a t(x)\,dx = \int_0^a t(t)\,dt\right]$

$I = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \log \cos x\, dx$ [Property IV]

$I = \frac{\pi}{2} \times \frac{-\pi}{2} \log 2$

$z = -\log 2$

68. (B) Given that

$\int_0^{\frac{\pi}{2}} \log \cos x dx = -\frac{\pi}{2} \log 2$

[Property IV]

$\int_0^{\frac{\pi}{2}} \log \sin x\, dx = -\frac{\pi}{2} \log 2$ **[solution – 70(*a*)]**

$\int_0^{\frac{\pi}{2}} \log \sin x\, dx = -\frac{\pi}{2} \log 2$

$\int_0^{\frac{\pi}{2}} \log \operatorname{cosec} x\, dx = \frac{\pi}{2} \log 2$

69. (C) I = $\int_0^{\frac{\pi}{2}} \log \tan x\, dx$ ...(i)

Property IV. $\int_0^a t(x) = \int_0^a t(a-x)\, dx$

$I = \int_0^{\frac{\pi}{2}} \log \cot x\, dx$ ...(ii)

by adding equation (i) and equation (ii)

$I + I = \int_0^{\frac{\pi}{2}} [\log \tan x + \log \cot x]\, dx$

$2I = \int_0^{\frac{\pi}{2}} \log(\tan x.\cot x)\, dx$

$2I = \int_0^{\frac{\pi}{2}} \log 1 dx$

$2I = \int_0^{\frac{\pi}{2}} 0 dx$

$I = 0$

70. (A) Given that

$\int_0^{\frac{\pi}{2}} \log \cos x\, dx = -\frac{\pi}{2} \log 2$

Property IV $\int_0^{\frac{\pi}{2}} \log \sin x dx = -\frac{\pi}{2} \log 2$

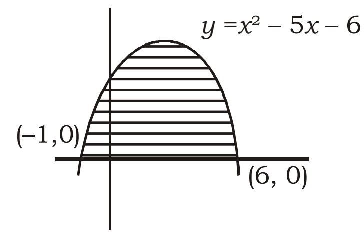
71. (D) $y = x^2 - 5x - 6$

curve cut the x-axis

then $y = 0$

$0 = x^2 - 5x - 6$

$x = -1, 6$



$x = -1, 6$

Area = $\int_{-1}^{6} y\, dx$

$= \int_{-1}^{6} (x^2 - 5x - 6)\, dx$

$= \left[\frac{x^3}{3} - \frac{5x^2}{2} - 6x\right]_{-1}^{6}$

$= \left(\frac{6^3}{3} - \frac{5+6^2}{2} - 6+6\right) - \left(\frac{(-1)^3}{3} - \frac{5(-1)^2}{2} - 6(-1)\right)$

On solving

Area = $-\frac{343}{6}$

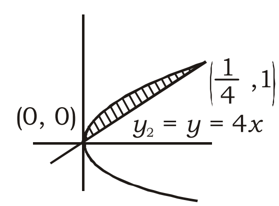
72. (A) curve $y^2 = 4x$ ...(i) (y_1)

line $y = 4x$ ...(ii) (y_2)

Solve the equations

$x = 0$ $gx = \frac{1}{4}$

$y = 0$ $y = 1$



Area = $\int_0^{\frac{1}{4}} (y_1 - y_2)\, dx$

$= \int_0^{\frac{1}{4}} (2\sqrt{x} - 4x)\, dx$

$= \left[2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4x^2}{2}\right]_0^{\frac{1}{4}}$

$= \left[\frac{4}{3}x^{\frac{3}{2}} - 2x^2\right]_0^{\frac{1}{4}}$

$= \left(\frac{4}{3}\left(\frac{1}{4}\right)^{\frac{3}{2}} - 2\times\left(\frac{1}{4}\right)^2\right) - (0-0)$

On solving

$= \frac{1}{24}$

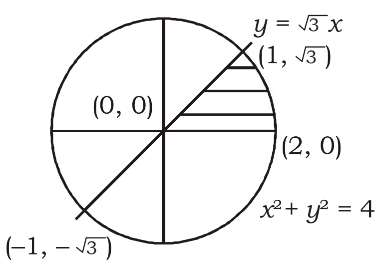
73. (C) Circle $x^2 + y^2 = 4$ ...(i)

line $y = \sqrt{3}\,x$ ...(ii)

Solve the equation

$x = -1 \quad x = 1$

$y = -\sqrt{3} \quad y - \sqrt{3}$



$y_1 = y = \sqrt{3}\,x$ line

$y_2 = y = \sqrt{4 - x^2}$ circle

Area $= \int_0^1 (y_1 - y_2)\,dx$

$= \int_0^1 \left(\sqrt{3}x - \sqrt{4-x^2}\right)dx$

$= \left[\frac{\sqrt{3}x^2}{2} - \left(\frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}\times 4\sin^{-1}\frac{x}{3}\right)\right]_0^1$

$= \left[\frac{\sqrt{3}}{2}\times 1 - \left(\frac{1}{2}\times 1\sqrt{4-1} + 2\sin^{-1}\frac{1}{2}\right)\right] -$

$\left[0 - \left(0 + 2\sin^{-1}0\right)\right]$

On solving.

$= \frac{-\pi}{3}$

74. (B) Curve $2x^2 + y^2 = 1$

$\frac{x^2}{\frac{1}{2}} + \frac{y^2}{1} = 1$

$a^2 = \frac{1}{2} \qquad b^2 = 1$

$a = \frac{1}{\sqrt{2}} \qquad b = 1$

Area of the ellipse $= \pi ab$

$= \pi \times \frac{1}{\sqrt{2}} \times 1$

$= \frac{\pi}{\sqrt{2}}$

75. (B) $\frac{dy}{dx} = \cos(x + y)$ ...(i)

Let $x + y = t$

differentiate both side

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

In equation (i)

$\frac{dt}{dx} - 1 = \cos t$

$\frac{dt}{dx} = 1 + \cos t$

$\frac{dt}{1 + \cos t} = dx$

$\frac{dt}{2\cos^2\frac{t}{2}} = dx$

$\frac{1}{2}\sec^2\frac{t}{2}\,dt = dx$

on integrating both side

$\int \frac{1}{2}\sec^2\frac{t}{2}\,dt = \int dx$

$\frac{1}{2}\cdot\frac{\tan\frac{t}{2}}{\frac{1}{2}} = x + c$

$\tan\frac{t}{2} = x + c$

$tan\left(\frac{x+y}{2}\right) = x + c$

76. (C) $\left(\frac{d^3y}{dx^3}\right)^{\frac{3}{4}} - 2 + \left(\frac{dy}{dx}\right)^2 + 5\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{2}} = 0$

$$\left(\frac{d^3y}{dx^2}\right)^{\frac{3}{4}} = 2 - \left(\frac{dy}{dx}\right)^2 - 5\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{2}}$$

$$\left(\frac{d^3y}{dx^2}\right)^3 = \left[2 - \left(\frac{dy}{dx}\right)^2 - 5\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{2}}\right]^4$$

order = 3

degree = 3

77. (D) $\frac{dy}{dx} + xy = x$

It is linear differential equation

$P = x$ $\qquad Q = x$

I.F = $e^{\int P dx}$

$= e^{\int x dx}$

$= e^{\frac{x^2}{2}}$

Hence general solution is

$y.(I.F.) = \int Q.(I.F)\, dx + c$

$y \times e^{\frac{x^2}{2}} = \int x \cdot e^{\frac{x^2}{2}}\, dx + c$

$y = 1 + c \cdot e^{-\frac{x^2}{2}}$

78. (B) Vertices (3, 0), (0, – 3), (k, 3)

Area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$18 = \frac{1}{2}\begin{vmatrix} 3 & 0 & 1 \\ 0 & -3 & 1 \\ k & 3 & 1 \end{vmatrix}$

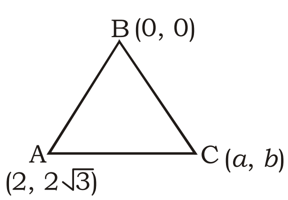
$36 = 3(-3 - 3) - 0\,(0 - 12) + 1\,(0 \times 3 - k \times (-3))$

$36 = -18 + 3k$

$54 = 3k$

$k = 18$

79. (A)



$AB = \sqrt{(2-0)^2 + (2\sqrt{3} - 0)^2}$

$AB = \sqrt{4 + 4 \times 3}$

$AB = \sqrt{16}$

$AB = 4$

$BC = \sqrt{(a-0)^2 + (b-0)^2}$

$BC = \sqrt{a^2 + b^2}$

$CA = \sqrt{(a-2)^2 + (b - 2\sqrt{3})^2}$

$CA = \sqrt{a^2 + 4 - 4a + b^2 + 12 - 4\sqrt{3}b}$

then $a^2 + b^2 = 16$ $\quad$ ...(i)

$a^2 + b^2 + 16 - 4a - 4\sqrt{3}\,b = 16$

$16 - 4a - 4a\sqrt{3}\,b = 0$

$4a + 4\sqrt{3}\,b = 16$

$4(a + \sqrt{3}\,b) = 4$

$a + \sqrt{3}\,b = 4$ $\quad$ ...(ii)

only (4, 0) satisfied the equation (i) and equation (ii)

i. e. $(a, b) = (4, 0)$

80. (C) Equation

$a_1 x^2 + 2h_1 xy + b_1 y^2 + 2g_1 x + c_1 = 0$

$a_1 = b_1$ $a_1 \neq 0$ $h_1 = 0$

81.(A) Area of circle $\pi r^2 = 36\pi$

$r = 6$

and centre = (3, 2)

Equation of circle

$(x-3)^2 + (y-2)^2 = (6)^2$

$x^2 + 9 - 6x + y^2 + 4 - 4y = 36$

$x^2 + y^2 - 6x - 4y + 13 - 36 = 0$

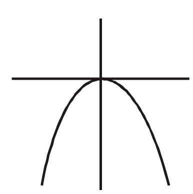
$x^2 + y^2 - 6x - 4y - 23 = 0$

82. (C) $x^2 = 8y$

On comparing with $x^2 = -4ay$

$4a = 8$

$a = 2$



Equation of directrix

$y = a$

$y = 2$

83. (D) Parabola $x^2 + 4x - y + 2 = 0$

$x^2 + 4x + 4 - 4 - y + 2 = 0$

$(x + 2)^2 - y - 2 = 0$

$(x + 2)^2 = y + 2$

$x + 2 = X \qquad y + 2 = Y$

$X^2 = Y$

On comparing with $x^2 = 4ay$

$4a = 1$

$a = \frac{1}{4}$

focus of the parabola $(0, a)$

$X = 0 \qquad Y = a$

$x + 2 = 0 \qquad y + 2 = \frac{1}{4}$

$x = -2 \qquad y = -\frac{7}{4}$

focus of parabola is $\left(-2, -\frac{7}{4}\right)$

84. (C) $[a\ b\ c] = \begin{vmatrix} 1 & 1 & -2 \\ 2 & -1 & 3 \\ -2 & -1 & 1 \end{vmatrix}$

where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{c} = -2\hat{i} - \hat{j} + \hat{k}$

$= 1(-1 + 3) - 1(2 + 6) - 2(-2 - 2)$

$= 1 \times 2 - 1 \times 8 - 2(-4)$

$= 2 - 8 + 8$

$[a\ b\ c] = 2$

85. (B) $\vec{a} \times (\vec{b} \times \vec{c}) = (a.c)\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$= [(\hat{i} + \hat{j} - 2\hat{k}) \cdot (-2\hat{i} - \hat{j} + \hat{k})\vec{b} - [(\hat{i} + \hat{j} - 2\hat{k}) \cdot (2\hat{i} - \hat{j} + 3\hat{k})]\vec{c}$

$= [-2 - 1 - 2]\vec{b} - [2 - 1 - 6]\vec{c}$

$= -5(2\hat{i} - \hat{j} + 3\hat{k}) + 5(-2\hat{i} - \hat{j} + \hat{k})$

$\vec{a} \times (\vec{b} \times \vec{c})$

$= -20\hat{i} - 10\hat{k}$

86. (B) $[a + b\ \ b + c\ \ c + a] = [a\ b\ c] + [b\ c\ a]$

$= [a\ b\ c] + [a\ b\ c]$

$= 2[a\ b\ c]$

$= 2 \times 2$

$= 4$

87. (B) Position vectors $5\hat{i} + l\hat{j}$, $2\hat{i} - 4\hat{j}$ and $\hat{i} - 5\hat{j}$ are collinear

then $\begin{vmatrix} 5 & \lambda & 1 \\ 2 & -4 & 1 \\ 1 & -5 & 1 \end{vmatrix} = 0$

$5(-4 + 5) - l(2 - 1) + 1(-10 + 4) = 0$

$5 - l - 6 = 0$

$-l - 1 = 0$

$l = -1$

88. (B) direction ratios (3, 0, 0) and (3, $-m$, 1) angle between two planes

$\cos q = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$

$\cos\frac{\pi}{4} = \left| \frac{9 - 0 \times m + 0 \times 1}{\sqrt{(3)^2 + (0)^2 + (0)^2}\sqrt{(3)^2 + (-m)^2 + (1)^2}} \right|$

$\frac{1}{\sqrt{2}} = \frac{9}{3\sqrt{10 + m^2}}$

$\frac{1}{\sqrt{2}} = \frac{3}{\sqrt{10 + m^2}}$

$\sqrt{10 + m^2} = 3\sqrt{2}$

$10 + m^2 = 9 \times 2$

$10 + m^2 = 18$

$m^2 = 8$

$m = \pm 2\sqrt{2}$

89. (A) $\frac{x - 1}{1} = \frac{y + 1}{-1} = \frac{z - 2}{-2}$

$a_1 = 1,\ b_1 = -1\ \ c_1 = -2$

and line $x - y - 2z = 0$

$a_2 = 1,\ b_2 = -1,\ c_2 = -2$

then $\sin q = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$\sin q = \frac{1 + 1 + 4}{\sqrt{1 + 1 + 4}\sqrt{1 + 1 + 4}}$

$\sin q = \frac{6}{\sqrt{6}\sqrt{6}}$

$\sin q = 1$

$q = \frac{\pi}{2}$

90. (C) $2^{(2-\log_2 6+\log_2 4)}$

$= 2^2 \cdot 2^{-\log_2 6} \cdot 2^{\log_2 4}$

$= 2^2 \cdot 2^{\log_2 \frac{1}{6}} \cdot 2^{\log_2 4}$

$= 2^2 \times \frac{1}{6} \times 4$

$= \frac{8}{3}$

91. (B) $\log_{10}2, \log_{10}(3^x - 1), \log_{10}(3^x + 3)$ are in A.P

$2b = a + c$

$2\log_{10}(3^x - 1) = \log_{10}2 + \log_{10}(3^x + 3)$

$\log_{10}(3^x - 1)^2 = \log_{10}[2 \times (3^x + 3)]$

$\log_{10}(3^x - 1)^2 = \log_{10}[2(3^x + 3)]$

On comparing

$(3^x - 1)^2 = 2(3^x + 3)$

$(3^x)^2 + 1 - 2 \cdot 2^x = 2\cdot 3^x + 6$

$(3^x)^2 - 4 \times 3^x - 5 = 0$

Let $3^x = y$

$y^2 - 4y - 5 = 0$

$y = 5$ $\quad$ $y = -1$

$3^x = 5$ $\quad$ $3^x = -1$

$x = \log_3 5$ $\quad$ $x = \log_3(-1)$ is not possible

92. (C) $\log_{10}\left(999 + \sqrt{x^2 - 5x + 7}\right) = 3$

$999 + \sqrt{x^2 - 5x + 7} = 10^3$

$\sqrt{x^2 - 5x + 7} = 1000 - 999$

$\sqrt{x^2 - 5x + 7} = 1$

$x^2 - 5x + 7 = 1$

$x^2 - 5x + 6 = 0$

$x = 2, 3$

93. (C) $\begin{bmatrix} 5 & 0 \\ 3 & -4 \end{bmatrix}^{-1} \begin{bmatrix} -x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$\begin{bmatrix} -x \\ y \end{bmatrix} = \uparrow \begin{bmatrix} -1 \\ 3 \end{bmatrix} \frac{2x}{x\left[\sqrt{a+x} + \sqrt{a-x}\right]}$

$\begin{bmatrix} -x \\ y \end{bmatrix} = [(-1 \times 5 + 3 \times 0) \;\; (-1 \times 3 + 3 + (-4))]$

$\begin{bmatrix} -x \\ y \end{bmatrix} = [-5, -15]$

$-x = -5$ $\quad$ $y = -15$

$x = 5$

94. (B) $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = k\, a^2b^2c^2$

$abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = k\, a^2b^2c^2$

$R_1 \to R_1 + R_2$

$\begin{vmatrix} 0 & 0 & 2c \\ a & -b & c \\ a & b & -c \end{vmatrix} = k\, abc$

$0 + 0 + 2c(ab + ab) = kabc$

$2c \times 2ab = kabc$

$4abc = kabc$

$k = 4$

95. (C) $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$= 1(1 - \log_z y \log_y z) - \log_x y (\log_y x - \log_z x \;\log_y z)$
$+ \log_x z (\log_y x \log_z y - \log_z x)$

$= 1(1-1) - (\log_x y \log_y x - \log_x y \log_y z \;\log_z x) +$
$\log_x z \;\log_z y \;\log_y x - \log_x z \;\log_z x$

$= 0 - \left(1 - \frac{\log y}{\log x} \times \frac{\log z}{\log y} \times \frac{\log x}{\log z}\right) + \frac{\log z}{\log x} \times$

$\frac{\log y}{\log z} \times \frac{\log x}{\log y} - 1$

$= -(1-1) + 1 - 1$

$= 0$

96. (D) $\left(\frac{x\sqrt{y}}{4} - \frac{2}{y\sqrt{x}}\right)^{14}$

total term = 14 + 1 = 15

middle term $= \frac{15+1}{2} = \frac{16}{2} = 8^{th}$ term

8^{th} term $T_8 = T_{7+1} = {}^{15}C_7 \left(\frac{x\sqrt{y}}{4}\right)^{15-7} \left(\frac{-2}{y\sqrt{x}}\right)^7$

$= -{}^{15}C_7 \frac{x^8y^4}{2^{16}} \times \frac{2^7}{y^7 x^{\frac{7}{2}}}$ x

$T_{7+1} = -{}^{15}C_7 \frac{x^{\frac{9}{2}} \cdot y^{-3}}{2^9}$

97. (B) $(3 + 4x)^6$

Total term = 6 + 1 = 7

middle term = $\frac{7+1}{2} = \frac{8}{2} = 4^{th}$ term

$T_4 = T_{3+1} = {}^6C_3\ (3)^3\ (4x)^3$

$= \frac{6!}{3!.3!} \times 27 \times 64x^3$

$= 20 \times 27 \times 64$

$= 34560$

98. (B) $\left(\frac{2}{3}x - \frac{3}{2x}\right)^n$

$T_4 = T_{3+1} = {}^nC_3 \left(\frac{2}{3}x\right)^{n-3} \left(\frac{-3}{2x}\right)^3$

$= {}^nC_3 \left(\frac{2}{3}\right)^{n-3} x^{n-3} \left(-\frac{3}{2}\right)^3 \left(\frac{1}{x}\right)^3$

$= {}^nC_3 \left(\frac{2}{3}\right)^{n-3} \left(\frac{-3}{2}\right)^3 x^{n-6}$

$= n - 6 = 0$

$n = 6$

99. (D) $\left(x^2 - \frac{1}{x}\right)^9$

Let T_{r+1} term is independent of x

$T_{r+1} = {}^9C_r\ (x^2)^{9-r} \left(-\frac{1}{x}\right)^r$

$= {}^9C_r\ x^{18-2r} (-1)^r\ x^{-r}$

$T_{r+1} = {}^9C_r\ r^{18-3r}\ (-1)^r$

Term independent of x

$18 - 3r = 0$

$r = 6$

Term $T_{6+1} = {}^9C_6\ (-1)^6$

$= \frac{9!}{6!3!} \times 1$

$T_{r+1} = 84$

100. (C) | 4 | 6 | 6 | {1, 2, 3, 4, 7, 9}

Only (1, 2, 3, 4) can put in first place

$4 \times 6 \times 6 = 24 \times 6 = 144$

101. (A) ${}^{15}C_{3r} = {}^{15}C_{r+3}$

$3r + (r + 3) = 15$

$4r + 3 = 15$

$4r = 12$

$r = 3$

102. (B) using (1, 2, 3, 4, 5, 6)

| 6 | 6 | 3 |

only (1, 3, 5)can put in this place for odd number

$6 \times 6 \times 3 = 36 \times 3$

$= 108$

103. (B) 6 men and 4 women

committee of 5 person

number of ways $= {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$

On solving

$= 6 \times 20 + 4 \times 15 + 1 \times 6$

$= 120 + 60 + 6$

$= 186$

104. (C) Total number of arrangement = 6! = 720

number of arrangement while all Hindi books are together = 4! × 3! = 144

number of arrangement while all Hindi books are not together = 720 – 144 = 576

105. (D) $\left(-\left(\sqrt{-1}\right)\right)^{8n+3} + \left(-\sqrt{-1}\right)^{12n+1}$

$(-i)^{8n+3} + (-i)^{12n+1}$

$(-i)^{8n} \cdot (-i)^3 + (-i)^{12n} \cdot (-i)^1$

$1 \times i + 1 \times (-i)$

$i + (-i) = 0$

106. (C) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

$\sin^{-1}\frac{5}{x} = \frac{\pi}{2} - \sin^{-1}\frac{12}{x}$

$\sin^{-1}\frac{5}{x} = \cos^{-1}\frac{12}{x}$

$\sin^{-1}\frac{5}{x} = \sin^{-1}\frac{\sqrt{x^2-144}}{x}$

$\frac{5}{x} = \frac{\sqrt{x^2-144}}{x}$

On squaring both side

$25 = x^2 - 144$

$x^2 = 169$

$x = 13$

107. (B) Girls Boys

25 50

35

15 : 10

3 : 2

number of girls = $\frac{3}{5} \times 100$

= 60

108. (B)

age	x	Person (f)	$t \times x$
0-10	5	7	35
10-20	15	15	225
20-30	25	20	500
30-40	35	35	1225
40-50	45	23	1035

$\Sigma f = 100 \quad \Sigma fx = 3020$

Mean = $\frac{\Sigma fx}{\Sigma f}$

= $\frac{3020}{100}$ = 30.2

109. (B) Agriculture = $\frac{90}{360} \times 72000$

A = 18000

Employment = $\frac{110}{360} \times 72000$

Employment = 22000

Industry = $\frac{50}{360} \times 72000$

***I* = 10000**

Miscellaneous = $\frac{70}{360} \times 72000$

M = 14000

Education = $\frac{40}{360} \times 72000$

***E* = 8000**

Maximum fund expend in Employment

110. (B) Education = 8000

111. (A) Employment =22000

I = 10000

Employment + Industry = 32000

112. (C) Education = 8000

Miscellaneous = 14000

M – E = 14000 – 8000 = 6000

113. (B) (2, 9, 3, 7, 5, 4, 3, 2, 10)

Arrange in ascending order

2, 2, 3, 3, 4, 5, 7, 9, 10

middle term = $\frac{9+1}{2} = \frac{10}{2}$ = 5th term

Median = 4

114.(B)

class	x
0-10	5
10-20	6
20-30	9
30-40	3
40-50	8

20-30 — modal class

$f_1 = 9, f_0 = 6, f_2 = 3$

$l_1 = 20, l_2 = 30$

Mode = $l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times (l_2 - l_1)$

= $20 + \frac{9-6}{2\times 9 - 6 - 3} \times (30–20)$

= $20 + \frac{3}{9} \times 10$

= $20 + \frac{10}{3}$

= 23.33

115. (B) $x^2 + y^2 + 6x + 9y + 5 = 0$

on comparing

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 63 \quad 2f = 9 \quad c = 5$

$g = 3 \; f = \frac{9}{2}$

Radius $r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{(3)^2 + \left(\frac{9}{2}\right)^2 - 5}$

$r = \frac{\sqrt{97}}{2}$

116. (C) (Major axis) $2a = 3 \times 2b$ (minor axis)

$a = 3b$

$b^2 = a^2(1 - e^2)$

$b^2 = (3b)^2 (1 - e^2)$

$b^2 = 9b^2 (1 - e^2)$

$\frac{1}{9} = 1 - e^2$

$e^2 = \frac{8}{9}$

$e = \frac{2\sqrt{2}}{3}$

117. (C) $y^2 = 16x$

$4a = 16$

$a = 4$

Let the point $(x_1 - y_1)$ on the parabola

$y_1^2 = 16x_1$ ...(i)

focal distance = $x_1 + a = 8$

$x_1 + 4 = 8$

$x_1 = 8$

$x_1 = 4$

From equation (i) $y_1 = \pm 8$

Cordinates on the parabola = (4, ± 8)

118. (C) $4x^2 + 16y^2 - 24x - 72y = 1$

$(2x)^2 - 24x + (4y)^2 - 32y = 1$

$(2x)^2 - 2 \times 2x \times 6 + 36 - 36 + (4y)^2 - 2 \times 4y \times 4 + 16 - 16 = 1$

$(2x - 6)^2 + (4y + 4)^2 - 52 = 1$

$4(x - 3)^2 + 16(y + 1)^2 = 53$

$$\frac{(x-3)^2}{\frac{53}{4}} + \frac{(y+1)^2}{\frac{53}{16}} = 1$$

$a^2 = \frac{53}{4}$ $b^2 = \frac{53}{16}$ where $a > b$

$b^2 = a^2(1 - e^2)$

$\frac{53}{16} = \frac{53}{4}(1 - e^2)$

$\frac{1}{4} = 1 - e^2$

$e^2 = 1 - \frac{1}{4}$

$e^2 = \frac{3}{4}$

$e = \frac{\sqrt{3}}{2}$

119. (B) $t(x) = \begin{cases} \frac{x^2 - 25}{x - 5} & \text{if } x \pm 5 \\ 3x + k & x = 5 \end{cases}$

$\lim_{x \to 5^+} t(x) = t(5)$

$\lim_{x \to 5} \frac{x^2 - 25}{x - 5} = 3 \times 5 + k$

$\lim_{x \to 5} \frac{(x-5)(x+5)}{(x-5)} = 15 + k$

$\lim_{x \to 5} (x + 5) = 15 + k$

$5 + 5 = 15 + k$

$k = 10 - 15$

$k = -5$

120. (D) $\lim_{x \to 0} \frac{\sqrt{a+x} - \sqrt{a-x}}{x}$ $\left[\frac{0}{0}\right]$ form

$= \lim_{x \to 0} \left[\frac{\sqrt{a+x} - \sqrt{a-x}}{x} \times \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}\right]$

$= \lim_{x \to 0} \frac{a + x - a + x}{x\left(\sqrt{a+x} + \sqrt{a-x}\right)}$

$= \lim_{x \to 0} \frac{2x}{x\left[\sqrt{a+x} + \sqrt{a-x}\right]}$

$= \frac{2}{\sqrt{a} + \sqrt{a}}$

$= \frac{2}{2\sqrt{a}}$

$= \frac{1}{\sqrt{a}}$

NDA (MATHS) MOCK TEST - 70 (Answer Key)

1.	(B)	21.	(D)	41.	(B)	61.	(D)	81.	(A)	101.	(A)
2.	(B)	22.	(B)	42.	(D)	62.	(A)	82.	(C)	102.	(B)
3.	(C)	23.	(B)	43.	(B)	63.	(A)	83.	(D)	103.	(B)
4.	(A)	24.	(D)	44.	(D)	64.	(C)	84.	(C)	104.	(C)
5.	(D)	25.	(D)	45.	(C)	65.	(D)	85.	(B)	105.	(D)
6.	(B)	26.	(C)	46.	(B)	66.	(B)	86.	(B)	106.	(C)
7.	(C)	27.	(C)	47.	(D)	67.	(C)	87.	(B)	107.	(B)
8.	(D)	28.	(C)	48.	(C)	68.	(B)	88.	(B)	108.	(B)
9.	(A)	29.	(B)	49.	(B)	69.	(C)	89.	(A)	109.	(B)
10.	(C)	30.	(D)	50.	(A)	70.	(A)	90.	(C)	110.	(B)
11.	(B)	31.	(D)	51.	(B)	71.	(D)	91.	(B)	111.	(A)
12.	(C)	32.	(D)	52.	(D)	72.	(A)	92.	(C)	112.	(C)
13.	(C)	33.	(D)	53.	(B)	73.	(C)	93.	(C)	113.	(B)
14.	(B)	34.	(D)	54.	(A)	74.	(B)	94.	(B)	114.	(B)
15.	(B)	35.	(B)	55.	(B)	75.	(B)	95.	(C)	115.	(B)
16.	(C)	36.	(A)	56.	(C)	76.	(C)	96.	(D)	116.	(C)
17.	(B)	37.	(B)	57.	(B)	77.	(D)	97.	(B)	117.	(C)
18.	(C)	38.	(D)	58.	(C)	78.	(B)	98.	(B)	118.	(C)
19.	(C)	39.	(B)	59.	(D)	79.	(A)	99.	(D)	119.	(B)
20.	(C)	40.	(B)	60.	(B)	80.	(C)	100.	(C)	120.	(D)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*