NDA MATHS MOCK TEST - 72 (SOLUTION)

1. (C) $\sin\left[\frac{\pi}{2}-\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right]$

$= \sin\left[\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)\right]$

$= \sin\left[\cos^{-1}\left(-\cos\frac{\pi}{4}\right)\right]$

$= \sin\left[\cos^{-1}\left(\cos\frac{3\pi}{4}\right)\right]$

$= \sin\left(\frac{3\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}$

2. (B) We know that

Var $(\lambda x) = \lambda^2 \text{Var}(x)$

data x = 2, 4, 5, 9, 11 and 19, var(x) = v

and y = 4, 8, 10, 14, 18, 22 and 38

$= 2x$

var(y) = var $(2x)$ = 2^2 var(x)

var(y) = $4v$

3. (A) Let (h, k, l) Locus of a point, which is equidistance from the points (1, 2, –3) and (1, –3, 2)

then

$\Rightarrow \sqrt{(h-1)^2+(k-2)^2+(l+3)^2} = \sqrt{(h-1)^2+(k+3)^2+(l-2)^2}$

$\Rightarrow (h-1)^2 + k^2 + 4 - 4k + l^2 + 9 + 6l = (h-1)^2 + k^2 + 9 + 6k + l^2 + 4 - 4l$

$\Rightarrow -4k + 6l = 6k - 4l$

$\Rightarrow 10l = 10k$

$\Rightarrow k = l \Rightarrow k - l = 0$

locus of the point is $y - z = 0$

4. (C) Line $4x = 3y = 6z$

$\frac{x-0}{\frac{1}{4}} = \frac{y-0}{\frac{1}{3}} = \frac{z-0}{\frac{1}{6}}$

$a_1 \quad b_1 \quad c_1$

and plane $2x - 3y + 3z = 7$

$a_2 = 2,\ b_2 = -3,\ c_2 = 3$

angle between line and plane

$\sin q = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$\sin q = \frac{\frac{1}{4}\times 2+\frac{1}{3}(-3)+\frac{1}{6}(3)}{\sqrt{\left(\frac{1}{4}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{6}\right)^2}\sqrt{(2)^2+(-3)^2+(3)^2}}$

$\sin q = 0$

$q = 0$

5. (D) Sphere $x^2 + y^2 + z^2 + 6x + 4y - 10z + 7 = 0$

compare with general equation

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + k = 0$

$u = 3,\ v = 2,\ w = -5$

and plane $2x - y + 2z + 4 = 0$

$a = 2,\ b = -1,\ c = 2,\ d = 4$

radius of sphere = $\left|\frac{-au-bv-cw+d}{\sqrt{a^2+b^2+c^2}}\right|$

$= \left|\frac{-2\times 3-(-1)\times 2-2\times(-5)+4}{\sqrt{(2)^2+(-1)^2+(2)^2}}\right|$

$r = \frac{10}{\sqrt{9}}$

$= \frac{10}{3}$

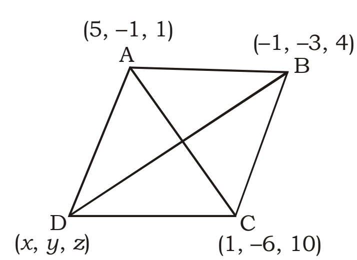
diameter of sphere = 2 × r

$= 2 \times \frac{10}{3}$

$= \frac{20}{3}$

6. (C) Let fourth vertex = (x, y, z)

In rhombus mid-points of the diagonals are same.



then

mid-point of AC = mid-point of BD

$$\left(\frac{5+1}{2}, \frac{-1-6}{2}, \frac{1+10}{2}\right) = \left(\frac{x-1}{2}, \frac{y-3}{2}, \frac{z+4}{2}\right)$$

$$\left(3, \frac{-7}{2}, \frac{11}{2}\right) = \left(\frac{x-1}{2}, \frac{y-3}{2}, \frac{z+4}{2}\right)$$

$$\frac{x-1}{2} = 3 \Rightarrow x = 7$$

$$\frac{y-3}{2} = \frac{-7}{2} \Rightarrow y = -4$$

$$\frac{z+4}{2} = \frac{11}{2} \Rightarrow z = 7$$

fourth vertex = (7, –4, 7)

7.(C) I. Statement :

$a \cdot \{(b + c) \times (a + b + c)\}$

$= a \cdot \{b \times a + b \times b + b \times c + c \times a + c \times b + c \times c\}$

$= a.\{b \times a + c \times a\}$

$= a.(b \times a) + a \cdot (c \times a)$

$= 0 + 0 = 0$

II. Statement : -

any three coplanar vectors d, e and f,

$(d \times e) \cdot f = 0$

Hence, both statements are correct.

8. (B) $x : y : z = 2 : \sqrt{3} : \sqrt{2}$

Let $x : 2k$, $y = \sqrt{3}\,k$, $z = \sqrt{2}\,k$

and given that $x\hat{i} + y\hat{j} + z\hat{k}$ is a unit vector.

$x^2 + y^2 + z^2 = 1$

$(2k)^2 + (\sqrt{3}\,k)^2 + (\sqrt{2}\,k)^2 = 1$

$9k^2 = 1$

$$k = \frac{1}{3}$$

then the value of $z = \sqrt{2}\,k$

$$z = \frac{\sqrt{2}}{3}$$

9. (A) Statement (S) :

$9x^2 - 16y^2 + 18x - 32y - 151 = 0$

$9x^2 + 18x - 16y^2 - 32y - 151 = 0$

$9(x + 1)^2 - 9 - 16(y + 1)^2 + 16 - 151 = 0$

$9(x + 1)^2 - 16(y + 1)^2 - 144 = 0$

$9(x + 1)^2 - 16(y + 1)^2 = 144$

$$\frac{(x+1)^2}{16} - \frac{(y+1)^2}{9} = 0$$

$a^2 = 16$, $b^2 = 9$

represents a hyperbola.

$$\left[\because \text{general equation } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\right]$$

Statement (S) is true.

Reason (R) is also true.

10. (A) I. Statement :

line $lx + my + n = 0$

$$y = -\frac{l}{m} - \frac{n}{m}$$

slope $m' = -\frac{l}{m}$ and $c = -\frac{n}{m}$

condition for normal

$$c = \pm \frac{m'(a^2 - b^2)}{\sqrt{a^2 + b^2 (m')^2}}$$

$$\frac{-n}{m} = \pm \frac{\frac{-l}{m}(a^2 - b^2)}{\sqrt{a^2 + \frac{b^2 l^2}{m^2}}}$$

$$\frac{n^2}{m^2} = \frac{\frac{l^2}{m^2}(a^2 - b^2)^2}{\left(a^2 + \frac{b^2 l^2}{m^2}\right)}$$

$$n^2\left(a^2 + \frac{b^2 l^2}{m^2}\right) = l^2 (a^2 - b^2)^2$$

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

(I) Statement is correct.

(II) Statement :- $m' = -\frac{l}{m}$, $c = \frac{-n}{m}$

Condition for tangent

$$c = \sqrt{a^2 (m')^2 + b^2}$$

$$\frac{-n}{m} = \sqrt{a^2\left(\frac{l^2}{m^2}\right) + b^2}$$

On squaring both side

$$\frac{n^2}{m^2} = \frac{a^2 l^2}{m^2} + b^2$$

$n^2 = a^2 l^2 + m^2 b^2$

(II) Statement is incorrect.

11. (D) Let $P(x, y)$

Definition of parabola

$PM^2 = SP^2$

$$\Rightarrow \left|\frac{4x-3y+2}{\sqrt{(4)^2+(-3)^2}}\right|^2 = (x+3)^2 + (y-2)^2$$

$$\Rightarrow \frac{16x^2+9y^2+4-24xy-12y+16x}{25} = x^2 + 9$$

$+ 6x + y^2 + 4 - 4y$

On solving

$\Rightarrow 9x^2 + 16y^2 + 24xy + 134x - 88y + 321 = 0$

12. (C) Hyperbola

$$\frac{(x+2)^2}{25} - \frac{(y-3)^2}{9} = 1$$

$a = 5,\ b = 3$

$$\frac{X^2}{25} - \frac{Y^2}{9} = 1 \text{ where } X = x + 2$$

$$Y = y - 3$$

parameter of hyperbola $(X, Y) = (a\sec\phi, b\tan\phi)$

$X = a\sec\phi,\quad Y = b\tan\phi$

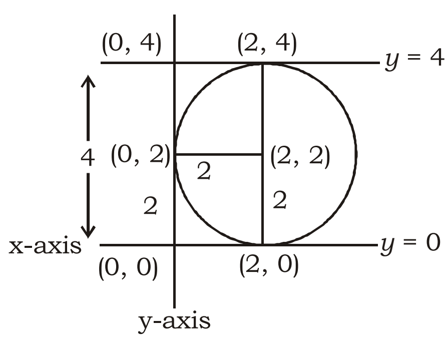
$x + 2 = a\sec\phi,\quad y - 3 = b\tan\phi$

$x = 5\sec\phi - 2,\quad y = 3\tan\phi + 3$

point on the hyperbola is

$(5\sec\phi - 2,\ 3\tan\phi + 3)$.

13. (A)



Radius of circle = 2

and centre = (2, 2)

then

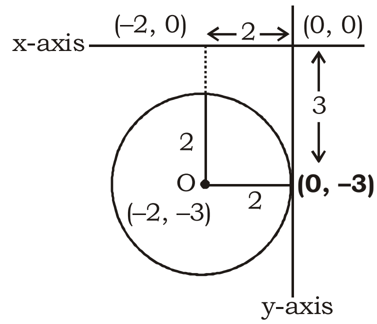
equation of circle

$(x - 2)^2 + (y - 2)^2 = (2)^2$

$x^2 + 4 - 4x + y^2 + 4 - 4y = 4$

$x^2 + y^2 - 4x - 4y + 4 = 0$

14. (D)



Equation of circle

$x^2 + y^2 + 4x + 6y + 9 = 0$

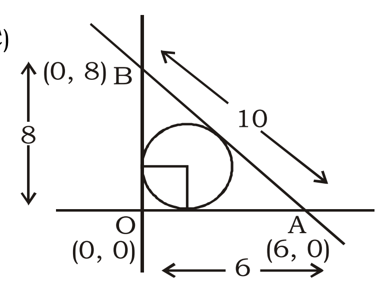
centre = (– 2, – 3)

and radius = $\sqrt{(-2)^2 + (-3)^2 - 9}$

= 2

point of contact = (0, – 3)

15. (C)



line $4x + 3y = 24$

$$\frac{x}{6} + \frac{y}{8} = 1$$

$OA = 6 = b$, $OB = 8 = a$, $AB = 10 = c$

$B(x_2, y_2) = (0, 8)$, $A(x_1, y_1) = (6, 0)$, $C(x_3, y_3) = (0,0)$

In centre of DOAB

$$\bar{x} = \frac{ax_1 + bx_2 + cx_3}{a+b+c}$$

$$\bar{x} = \frac{8\times6+6\times0+10\times0}{8+6+10}$$

$$\bar{x} = \frac{48}{24} = 2$$

$$\bar{y} = \frac{ay_1 + by_2 + cy_3}{a+b+c}$$

$$= \frac{8\times0+6\times8+10\times0}{8+6+10}$$

$$\bar{y} = \frac{48}{24} = 2$$

In centre $(\bar{x}, \bar{y}) = (2, 2)$

16. (A) lines $3x-2y+5=0$ and $6x-4y-11=0$

slope $m_1 = \frac{3}{2}$, $m_2 = \frac{3}{2}$

angle between lines

$$\tan\theta = \frac{m_1 - m_2}{1+m_1 m_2}$$

$$\tan q = \frac{\frac{3}{2}-\frac{3}{2}}{1+\frac{3}{2}\times\frac{3}{2}}$$

$\tan q = 0$

$q = 0$

17. (B) Lines $3x-2y+5=0$ and $6x-4y-11=0$

$d_1 = 5$ $\quad 3x-2y-\frac{11}{2}=0$

$d_2 = -\frac{11}{2}$

distance between lines

$$D = \left|\frac{d_1-d_2}{\sqrt{a^2+b^2}}\right|$$

$$D = \left|\frac{5-\left(-\frac{11}{2}\right)}{\sqrt{(3)^2+(2)^2}}\right|$$

$$D = \frac{21}{2\sqrt{13}}$$

18.(C) Line $3x-2y+5=0$

Slope $m_1 = \frac{3}{2}$

equation of the line which is perpendicular the given and passing through point (–2, 3)

slope of line = $m = \frac{-1}{m_1} = \frac{-2}{3}$

equation $y-y_1 = m(x-x_1)$

$$y-3 = -\frac{2}{3}(x+2)$$

$2x+3y=5$

19. (D) Line $6x-4y-11=0$

Slope $m = \frac{3}{2}$

equation of the line which is parallel to the given line and passing through the point (– 3, 0)

$y-y_1 = m(x-x_1)$

$$y-0 = \frac{3}{2}(x+3)$$

$3x-2y+9=0$

20. (A) $x^2+y^2+4x-2y-4=0$

point (3, 2)

$(3)^2+(2)^2+4\times3-2\times2-4$

$9+4+12-4-4$

$17>0$

exterior point

21.(C) lines $3x+2y+5=0$, $2x-y+k=0$ and $-x+y+3=0$ are concurrent.

condition of concurrency

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3 & 2 & 5 \\ 2 & -1 & k \\ -1 & 1 & 3 \end{vmatrix} = 0$$

$3(-3-k)-2(6+k)+5(2-1)=0$

on solving

$$k = \frac{-16}{5}$$

22. (C) Points (a, ma), $\{b, (m+1)b\}$ and $\{c, (m+2)c\}$ are collinear.

condition

$$\begin{vmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} a & ma & 1 \\ b & (m+1)b & 1 \\ c & (m+2)c & 1 \end{vmatrix} = 0$$

$a[(m+1)b-(m+2)c] - ma[b-c] + 1[(m+2)bc - (m+1)bc] = 0$

$ab - 2ac + bc = 0$

$ab + bc = 2ac$

$b(a+c) = 2ac$

$$b = \frac{2ac}{a+c}$$

Hence a, b, c are in H.P for all m.

23. (D) Differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + \left(\frac{dy}{dx}\right)^5 + 4\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{4}} - 5y = 0$$

$$\left(\frac{d^3y}{dx^3}\right)^2 = \left[5y - \left(\frac{dy}{dx}\right)^5 - 4\left(\frac{d^2y}{dx_2}\right)^{\frac{3}{4}}\right]^3$$

order = 3 and degree = 2

24. (C) $(1 + e^x)y\,dy = e^x dx$

$$y\,dy = \frac{e^x}{1+e^x}\,dx$$

On integrating both side

$$\int y dy = \int \frac{e^x}{1+e^x}\,dx$$

Let $1 + e^x = t$

$e^x dx = dt$

$$\frac{y^2}{2} = \int \frac{dt}{t}$$

$$\frac{y^2}{2} = \log t + \log c$$

$$\frac{y^2}{2} = \log(1 + e^x) + \log c$$

$$\frac{y^2}{2} = \log\{c\,(1 + e^x)\}$$

$y^2 = \log\{c(1 + e^x)\}^2$

25. (D) $\frac{dy}{dx} = 1 - 2x + y - 2xy$

$$\frac{dy}{dx} = 1(1 - 2x) + y(1 - 2x)$$

$$\frac{dy}{dx} = (1 - 2x)\,(1 + y)$$

$$\frac{dy}{1+y} = (1 - 2x)dx$$

On integrating both side

$$\int \frac{dy}{1+y} = \int (1-2x)\,dx$$

$$\log\,(1 + y) = x - \frac{2x^2}{2} + c$$

$\log(1+ y) = x - x^2 + c$ ...(i)

at $y(1) = 0 \Rightarrow x = 1,\ y = 0$

from equation (i)

$\log(1) = + 1 - 1 + c$

$0 = + c$

$c = 0$

from equation (i)

$\log\,(1 + y) = x - x^2$

$1 + y = e^{x - x^2}$

$y = e^{x(1-x)} - 1$

26. (B) Curve $\sqrt{x} + \sqrt{y} = 4$

$$\sqrt{y} = 4 - \sqrt{x}$$

$$y = 16 + x - 8\sqrt{x}$$

curve cuts y-axis at $x = 0$ and $y = 16$

curve cuts x-axis at $y = 0$ and $x = 16$

$$\text{Area} = \int_0^{16} y.dx$$

$$= \int_0^{16}\left[16 + x - 8x^{\frac{1}{2}}\right]dx$$

$$= \left[16x + \frac{x^2}{2} - 8x^{\frac{3}{2}} \times \frac{2}{3}\right]_0^{16}$$

$$= \left[256 + 128 - \frac{16}{3}\times 64 - 0\right]$$

$$= \frac{128}{3}\ \text{sq. unit}$$

27. (C) $I = \int \frac{(1+\log x)}{\sin^2(x\log x)}\,dx$

Let $x\log x = t$

$(1 + \log x)dx = dt$

$$I = \int \frac{dt}{\sin^2 t}$$

$$I = \int \text{cosec}^2 t\ dt$$

$$I = -\cot t + C$$

$$I = -\frac{1}{\tan t}$$

$$I = -\frac{1}{\tan(x\log x)} + C$$

28. (A) Formula

$$\int \text{cosec}\ x\,dx = \log|\text{cosec}\,x - \cot x| + C$$

$$= \log\left|\tan\frac{x}{2}\right| + C$$

29. (B) $I = \int_0^{\frac{\pi}{2}} \sin 2x \log \tan x \, dx$ ...(i)

$I = \int_0^{\frac{\pi}{2}} \sin(\pi - 2x) \log \tan\left(\frac{\pi}{2} - x\right) dx$

(Property IV)

$I = \int_0^{\frac{\pi}{2}} \sin 2x \log \cot x \, dx$ ...(ii)

On adding equation (i) and equation (ii)

$2I = \int_0^{\frac{\pi}{2}} \sin 2x \, [\log \tan x + \log \cot x] dx$

$2I = \int_0^{\frac{\pi}{2}} \sin 2x \, (\log 1) dx$

$2I = 0$

$I = 0$

30. (B) $\int_{\log 2}^{x} (e^x - 1)^{-1} dx = \log \frac{3}{2}$

$\int_{\log 2}^{x} \frac{1}{e^x - 1} dx = \log \frac{3}{2}$

$\int_{\log 2}^{x} \frac{e^{-x}}{(1 - e^{-x})} dx = \log \frac{3}{2}$

Let $1 - e^{-x} = t$ $\quad x \to \log 2, \quad t \to \frac{1}{2}$

$-e^{-x}(-1) dx = dt$ $\quad x \to x, \quad t \to t$

$e^{-x} dx = dt$

$\int_{\frac{1}{2}}^{t} \frac{dt}{t} = \log \frac{3}{2}$

$[\log t]_{\frac{1}{2}}^{t} = \log \frac{3}{2}$

$\log t - \log \frac{1}{2} = \log \frac{3}{2}$

$\log t = \log \frac{3}{2} + \log \frac{1}{2}$

$t = \frac{3}{4}$

$1 - e^{-x} = \frac{3}{4}$

$e^{-x} = \frac{1}{4}$

$-x = \log \frac{1}{4}$

$x = \log 4$

31. (B) $I = \int_{-3}^{2} [x] \, dx$

where [ ] is greatest Integer.

$I = \int_{-3}^{-2} [x] \, dx + \int_{-2}^{-1} [x] \, dx + \int_{-1}^{0} [x] \, dx + \int_{0}^{1} [x] \, dx$

$+ \int_{1}^{2} [x] \, dx$

$I = \int_{-3}^{-2} (-3) \, dx + \int_{-2}^{-1} (-2) \, dx + \int_{-1}^{0} (-1) \, dx$

$+ \int_{0}^{1} (0) \, dx + \int_{1}^{2} 1.dx$

$I = -3[x]_{-3}^{-2} - 2[x]_{-2}^{-1} + (-1)[x]_{-1}^{0} + 0 + [x]_{1}^{2}$

$I = -3[-2 + 3] - 2[-1 + 2] - [0 + 1] + [2 - 1]$

$= -3 - 2 - 1 + 1$

$= -5$

32. (B) $I = \int_0^{\pi} \frac{\cos^3 x}{x^2 + (\pi - x)^2} dx$ ...(i)

$I = \int_0^{\pi} \frac{\cos^3(\pi - x)}{(\pi - x)^2 + x^2} dx$ [Property- IV]

$I = \int_0^{\pi} \frac{-\cos^3 x}{x^2 + (\pi - x)^2} dx$ ...(ii)

on adding equation (i) and equation (ii)

$2I = \int_0^{\pi} 0 \, dx$

$I = 0$

33. (C) $f(x) = \log(x) - 2x$ ...(i)

Differentiate *w.r.t.* 'x'

$f'(x) = \frac{1}{x} - 2$ ...(ii)

again differentiate *w.r.t.* 'x'

$f''(x) = \frac{-1}{x^2}$ ...(iii)

for maxima and minima

$f'(x) = 0$

$\frac{1}{x} - 2 = 0$

$\frac{1}{x} = +2$

$x = \frac{1}{2}$

on putting $x = \frac{1}{2}$ in equation (iii)

$f''\left(\frac{1}{2}\right) = -4 < 0$ (maxima)

maximum value of function $f(x)$ $\left(\text{at } x = \frac{1}{2}\right)$

$= \log\left(\frac{1}{2}\right) - 2 \times \frac{1}{2}$

$= -\log 2 - 1$

34. (A) $f(x) = x^3 - 3x^2 - 9x + 8$ ...(i)

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$ ...(ii)

for maxima and minima

$f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$3[x^2 - 2x - 3] = 0$

$(x + 1)(x - 3) = 0$

$x = -1, \ x = 3$

on putting $x = -1$ in equation (ii)

$f''(-1) = 5 \times -1 - 6 = -12 < 0$ (maxima)

on putting $x = 3$ in equation (ii)

$f''(3) = 6 \times 3 - 6$

$= 12 > 0$ (minima)

minimum value of $f(x)$ (at $x = 3$)

$= 3^3 - 3 \times 3^2 - 9 \times 3 + 8$

$= -19$

35. (D) $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots}}}$

$y = \sqrt{\cos x + y}$

$y^2 = \cos x + y$

Differentiate both side *w.r.t.* 'x'

$2y \frac{dy}{dx} = -\sin x + 1$

$2y \frac{dy}{dx} = \frac{1 - \sin x}{2y}$

36. (A) $f(x) = \log x$

$f(\log x) = \log(\log x)$

differentiate *w.r.t.* 'x'

$f'(\log x) = \frac{1}{\log x} \times \frac{1}{x}$

$f'(\log x) = (x \log x)^{-1}$

37. (B) $y = \log\sqrt{\sec x}$

differentiate *w.r.t.* 'x'

$\frac{dy}{dx} = \frac{1}{\sqrt{\sec x}} \cdot \frac{1}{2\sqrt{\sec x}}$ $(\sec x + \tan x)$

$\frac{dy}{dx} = \frac{\sec x + \tan x}{2\sec x}$

$\lim_{x \to 0^-}\left(\frac{dy}{dx}\right)_{at\, x = \frac{\pi}{3}} = \frac{\sec\frac{\pi}{3} + \tan\frac{\pi}{3}}{2\sec\frac{\pi}{3}}$

$= \frac{2 + \sqrt{3}}{2 \times 2}$

$= \frac{2 + \sqrt{3}}{4}$

38. (A) $x = 2y\sqrt{1 + y^2}$

Differentiate w.r.t 'y'

$\frac{dx}{dy} = 2\left[y.\frac{1 \times 2y}{2\sqrt{1 + y^2}} + \sqrt{1 + y^2}.1\right]$

$\frac{dx}{dy} = 2\left[\frac{y^2}{\sqrt{1 + y^2}} + \sqrt{1 + y^2}\right]$

$\frac{dx}{dy} = 2\left[\frac{1 + 2y^2}{\sqrt{1 + y^2}}\right]$

$\frac{dx}{dy} = \frac{1}{2}\frac{\sqrt{1 + y^2}}{1 + 2y^2}$

39. (D) $f(x) = \begin{cases} 3x^2 - 2, -1 \le x < 2 \\ 5 \quad , \quad x = 2 \\ 3x^2 + 5x, x > 2 \end{cases}$

for $x = 5$

$f(x) = 3x^2 + 5x$

$f'(x) = 3 \times 2x + 5$

$= 6x + 5$

$f'(5) = 6 \times 5 + 5$

$f'(5) = 35$

40. (D) $\lim_{x \to 0} f(x) = \lim_{x \to 0} e^{-\frac{1}{x}}$

L.H.L. $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h)$

$= \lim_{h \to 0} e^{\frac{-1}{-h}}$

$= \lim_{h \to 0} e^{\frac{1}{h}}$

$= e^{\frac{1}{0}}$

$= e^{\infty} = \infty$

R.H.L. = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} e^{\frac{-1}{h}}$

$= \lim_{h \to 0} e^{\frac{-1}{h}}$

$= e^{\frac{-1}{0}}$

$= e^{-\infty} = 0$

L.H.L. ≠ R.H.L.

limits does not exist.

41. (C) $f(x) = \begin{cases} \dfrac{x^2 - 5x + 6}{(x-2)} &, x \neq 2 \\ \lambda &, x = 2 \end{cases}$ is continuous

at $x = 2$

then

$\lim_{x \to 2^+} f(x) = f(2) = \lim_{x \to 2^-} f(x)$

$\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{(x-2)} = \lambda$

$\lim_{x \to 2} \dfrac{(x-2)(x-3)}{(x-2)} = \lambda$

$\lim_{x \to 2} (x - 3) = \lambda$

$-1 = \lambda$

$\lambda = -1$

42. (C) $f(x) = \sin^{-1} (\log_5 2x)$

$-1 \leq \log_5 2x \leq 1$

$5^{-1} \leq 2x \leq 5^1$

$\dfrac{1}{5} \leq 2x \leq 5$

$\dfrac{1}{10} \leq x \leq \dfrac{5}{2}$

domain of $f(x) = \left[\dfrac{1}{10}, \dfrac{5}{2}\right]$

43. (B) Given that $g(x) = x^3 + 2x^2 - 5x + 2$

and $f'(x) = g'(x)$

On integrating both side

$f(x) = g(x) + C$

$f(x) = x^3 + 2x^2 - 5x + 2 + c$ ...(i)

given that $f(-1) = 3$

$3 = -1 + 2 + 5 + 2 + c$

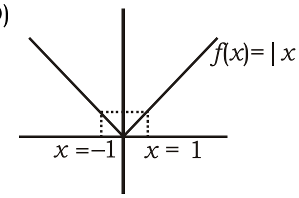
$c = -5$

from equation (i)

$f(x) = x^3 + 2x^2 - 5x + 2 - 5$

$f(x) = x^3 + 2x^2 - 5x - 3$

44. (D)



$f(x) = |x|$

$x = 1, f(1) = 1$

$x = -1\ f(-1) = 1$

Function is not one-one function.

The value of $f(x)$ will always positive and i.e. i.e.$f(x)$ is not onto function.

Then function neither one-one nor onto.

45.(D) Equation $5 f(x) + 3f\left(\dfrac{x+39}{x-1}\right) = 5x + 15$

at $x = 6$

$5f(6) + 3f(9) = 45$ ...(i)

at $x = 9$

$5f(9) + 3f(6) = 60$ ...(ii)

equation (i) × 5 – equation (ii) × 3

$25f(6) - 9f(6) = 225 - 180$

$16f(6) = 45$

$f(6) = \dfrac{45}{16}$

46. (B) $\sin^{-1}\left[\sin \dfrac{5\pi}{4}\right]$

$= \sin^{-1}\left[\sin\left(\pi + \dfrac{\pi}{4}\right)\right]$

$= \sin^{-1}\left[-\sin \dfrac{\pi}{4}\right]$

$= \sin^{-1}\left[\sin\left(-\dfrac{\pi}{4}\right)\right]$

$= -\dfrac{\pi}{4}$

47. (A)$\Rightarrow$ $\sec\left[\text{cosec}^{-1}\left(\text{cosec}\ \frac{7\pi}{4}\right)\right]$

$\Rightarrow \sec\left[\text{cosec}^{-1}\left(\text{cosec}\ \left(2\pi - \frac{\pi}{4}\right)\right)\right]$

$\Rightarrow \sec\left[\text{cosec}^{-1}\left(-\text{cosec}\ \left(\frac{\pi}{4}\right)\right)\right]$

$\Rightarrow \sec\left[\text{cosec}^{-1}\left(\text{cosec}\ \left(-\frac{\pi}{4}\right)\right)\right]$

$\Rightarrow \sec\left(-\frac{\pi}{4}\right) = \sec\frac{\pi}{4} = \sqrt{2}$.

48. (C) $\sin^{-1} x + \sin^{-1} y = \pi$

$\frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} y = \text{p}$

$-\cos^{-1} x - \cos^{-1} y = 0$

$\cos^{-1} x + \cos^{-1} y = 0$

49. (D) $\cot\left[\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$

$= \cot\left[\frac{1}{2}\times 2\tan^{-1} x + \frac{1}{2}\times 2\tan^{-1} x\right]$

$\left[\because \tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}\right]$

$= \cot\ [2\tan^{-1} x]$

$= \cot\left[\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$= \cot\left[\cot^{-1}\left(\frac{1-x^2}{2x}\right)\right]$

$= \frac{1-x^2}{2x}$

50. (A) Total Sample (S) = {(TTT), (TTH), (THT), (HTT), (HHT), (THH), (HHH), (HTH)}

n(s) = 2^3 = 8

E = {(HHH), (HHT), (THH), (HTH), (TTH), (THT), (HTT)}

$P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$

51. (C) Total cards $n(S)$ = 52

there are 4 ace and 13 spade cards in pack of 52 cards.

1 card (ace of spade) common in 4 ace and 13 spade.

So, n(E) = 3 + 13 = 16

$P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$

52. (B) A leap year = 366 days

= 52 weeks and 2 days

2 days arranged in this like -

(Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday) (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday) (Sunday, Monday).

Total sample space $n(S)$ = 7

$n(E)$ = 2

$P(E) = \frac{n(E)}{n(S)}$

$= \frac{2}{7}$

53. (C) UNIVERSAL

Vowel = U, I, E, A

(UIEA) NVRSL

As a one letter

(UIEA)

Total arrangement = 6! × 4!

= 720 × 24

= 17280

54.(D) 8 boys in two chairs

number of ways = ${}^8P_2$

$= \frac{8!}{6!}$

$= \frac{8\times 7\times 6!}{6!}$

= 56

55. (A) Total number of cricket players = 15

In a team captain will be always same.

number of ways = ${}^{15-1}C_{11-1}$ = ${}^{14}C_{10}$

$= \frac{14!}{10!4!}$

= 1001

56. (B) $\left(-3x^3 + \frac{1}{4x^3}\right)^6$

4th term = $T_4 = T_{3+1} = {}^6C_3\ (-3x^3)^3.\ \left(\frac{1}{4x^3}\right)^3$

$= {}^6C_3\,(-27)x^9 \times \dfrac{1}{64x^9}$

$= -\dfrac{6!}{3!3!} \times \dfrac{27}{64}$

$= -\dfrac{135}{16}$

57. (B) Binomial theorem

$(1+x)^n = {}^nC_0\,x^0 + {}^nC_1x^1 + {}^nC_2\,x^2 + ... + {}^nC_n\,x^n$

On putting $n = 7$ and $x = 1$

$\Rightarrow$ $(1+1)^7 = {}^7C_0(1)^0 + {}^7C_1\,(1)^1 + {}^7C_2\,(1)^2 + {}^7C_3(1)^3 + {}^7C_4\,(1)^4 + {}^7C_5\,(1)^5 + {}^7C_6\,(1)^6 + {}^7C_7(1)^7$

$\Rightarrow$ ${}^7C_0 + {}^7C_1 + {}^7C_2 + {}^7C_3 + {}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7$

$= 2^7 = 128$

58. (C) general term in the expansion of $\left(ax^2 - \dfrac{1}{bx}\right)^{12}$

$T_{r+1} = {}^{12}C_r\,(ax^2)^{12-r}\left(\dfrac{-1}{bx}\right)^r$

$= {}^{12}C_r\;a^{12-r}\left(\dfrac{-1}{b}\right)^r x^{24-3r}$

then $24 - 3r = -6$

$30 = 3r$

$r = 10$

$T_{11} = {}^{12}C_{10}\;a^2\left(\dfrac{-1}{b}\right)^{10} x^{-6}$

then coefficient of $x^{-6} = {}^{12}C_{10}\;\dfrac{a^2}{b^{10}}$

$= \dfrac{66a^2}{b^{10}}$

59. (D) Assertion (A) : -

$= C(24, 4) + \sum_{r=0}^{4} C(28-r, 3)$

$= {}^{24}C_4 + C(28-0, 3) + C(28-1, 3) + C(28-2, 3) + C(28-3, 3) + C(28-4, 3)$

$= {}^{24}C_4 + {}^{28}C_3 + {}^{27}C_3 + {}^{26}C_3 + {}^{25}C_3 + {}^{24}C_3$

$= {}^{24}C_3 + {}^{24}C_4 + {}^{25}C_3 + {}^{26}C_3 + {}^{27}C_3 + {}^{28}C_3$

$= {}^{25}C_4 + {}^{25}C_3 + {}^{26}C_3 + {}^{27}C_3 + {}^{28}C_3$

$[\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}]$

$= {}^{26}C_4 + {}^{26}C_3 + {}^{27}C_3 + {}^{28}C_3$

$= {}^{27}C_4 + {}^{27}C_3 + {}^{28}C_3$

$= {}^{28}C_4 + {}^{28}C_3$

$= {}^{29}C_4$

Assertion (A) is incorrect.

Reason (R) is correct.

60. (B) $(1 + x + x^2 + x^3)6$

$= (1+x)^6\,(1+x^2)^6$

$= (1 + {}^6c_1x + {}^6c_2x^2 + {}^6c_3x^3 ...)\,(1 + {}^6c_1x^2 + {}^6c_2x^4 + ...)$

coefficient of $x^2 = {}^6c_1 + {}^6c_2$

$= 6 + \dfrac{6!}{2!4!}$

$= 6 + 15$

$= 21$

61.(C) Determinant $\begin{vmatrix} 5x-3y & 5x-4y & 5x-5y \\ 5x-2y & 5x-3y & 5x-4y \\ 5x-6y & 5x-7y & 5x-8y \end{vmatrix}$

$C_1 \to C_1 - C_2,\ C_3 \to C_3 - C_2$

$= \begin{vmatrix} y & 5x-4y & -y \\ y & 5x-3y & -y \\ y & 5x-7y & -y \end{vmatrix}$

$= -\begin{vmatrix} y & 5x-4y & y \\ y & 5x-3y & y \\ y & 5x-7y & y \end{vmatrix}$

$= 0$ [$\because$ two columns are identical.]

62. (D) $\begin{vmatrix} a+b & c & c-a \\ b+c & a & a-b \\ c+a & b & b-c \end{vmatrix}$

$C_1 \to C_1 + C_2$ and $C_3 \to C_3 - C_2$

$= \begin{vmatrix} a+b+c & c & -a \\ a+b+c & a & -b \\ a+b+c & b & -c \end{vmatrix}$

$= -(a+b+c)\begin{vmatrix} 1 & c & a \\ 1 & a & b \\ 1 & b & c \end{vmatrix}$

$= -(a+b+c)\,[1(ac-b^2) - c\,(c-b) + a\,(b-a)]$

$= -(a+b+c)\,[ac - b^2 - c^2 + bc + ab - a^2]$

$= (a+b+c)[ac - b^2 - c^2 + ab + ab - a^2]$

$= (a+b+c)[a^2 + b^2 + c^2 - ab - bc - ca]$

$= a^3 + b^3 + c^3 - 3abc$

$[\because a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)]$

63. (B) $\Delta_1 = \begin{vmatrix} x & a & a \\ b & x & a \\ b & b & x \end{vmatrix}$

$C_2 \to C_2 - C_3$

$\Delta_1 = \begin{vmatrix} x & 0 & a \\ b & x-a & a \\ b & b-x & x \end{vmatrix}$

$\Delta_1 = x(x^2 - ax - ab + ax) + 0 + a(b^2 - bx - bx + ab)$

$\Delta_1 = x(x^2 - ab) + a(b^2 - 2bx + ab)$

$\Delta_1 = x^3 - abx + ab^2 - 2abx + a^2b$

$\Delta_1 = x^3 - 3abx + ab^2 + a^2b$

$\frac{d}{dx}(\Delta_1) = 3x^2 - 3ab$ and $\Delta_2 = \begin{vmatrix} x & a \\ b & x \end{vmatrix}$

$= 3(x^2 - ab)$ $\qquad \Delta_2 = x^2 - ab$

$\frac{d}{dx}(\Delta_1) = 3\Delta_2$

64. (B) $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 2 & 4 \\ 2 & 5 & 1 \end{bmatrix}$

$|A| = 1(2 - 20) + 2(2 - 8) + 3(10 - 4)$

$= -18 - 12 + 18$

$= -12 \neq 0$

co-factors of A

$C_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix}, C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 4 \\ 2 & 1 \end{vmatrix}, C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 2 \\ 2 & 5 \end{vmatrix}$

$= -18 \qquad = 6 \qquad = 6$

$C_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 3 \\ 5 & 1 \end{vmatrix}, C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}, C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -2 \\ 2 & 5 \end{vmatrix}$

$= 17 \qquad = -5 \qquad = -9$

$C_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 3 \\ 2 & 4 \end{vmatrix}, C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}, C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ 2 & 2 \end{vmatrix}$

$= -14 \qquad = 2 \qquad = 6$

$C = \begin{bmatrix} -18 & 6 & 6 \\ 17 & -5 & -9 \\ -14 & 2 & 6 \end{bmatrix}$

adj A = transpose of C = $\begin{bmatrix} -18 & 17 & -14 \\ 6 & -5 & 2 \\ 6 & -9 & 6 \end{bmatrix}$

$A^{-1} = \frac{adj\,A}{|A|}$

$A^{-1} = \begin{bmatrix} \frac{3}{2} & \frac{-17}{12} & \frac{7}{6} \\ \frac{-1}{2} & \frac{5}{12} & \frac{-1}{6} \\ \frac{-1}{2} & \frac{3}{4} & \frac{-1}{2} \end{bmatrix}$

65. (C) $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 2 & 4 \\ 2 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & -1 \\ 0 & 3 & 2 \end{bmatrix}$

$|B - A| = \begin{bmatrix} 0 & 4 & 1 \\ -4 & 1 & -5 \\ -2 & -2 & 1 \end{bmatrix}$

$= 0 - 4(-4 - 10) + 1(8 + 2)$

$= -4(-14) + 10$

$= +56 + 10$

$= 66$

66. (A) $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 2 & 4 \\ 2 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & -1 \\ 0 & 3 & 2 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & -1 \\ 0 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & -2 & 3 \\ 2 & 2 & 4 \\ 2 & 5 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+4+8 & -2+4+20 & 3+8+4 \\ -2+6-2 & 4+6-5 & -6+12-1 \\ 0+6+4 & 0+6+10 & 0+12+2 \end{bmatrix}$

$= \begin{bmatrix} 13 & 22 & 15 \\ 2 & 5 & 5 \\ 10 & 16 & 14 \end{bmatrix}$

67. (D) $B = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & -1 \\ 0 & 3 & 2 \end{bmatrix}$

co-factors of B

$C_{11} = (-1)^{1+1}\begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix}, C_{12} = (-1)^{1+2}\begin{vmatrix} -2 & -1 \\ 0 & 2 \end{vmatrix}, C_{13} = (-1)^{1+3}\begin{vmatrix} -2 & 3 \\ 0 & 3 \end{vmatrix}$

$= 9 \qquad = 4 \qquad = -6$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix}, C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix}, C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix}$

$= 8 \qquad = 2 \qquad = -3$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 4 \\ 3 & -1 \end{vmatrix}, C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 4 \\ -2 & -1 \end{vmatrix}, C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix}$

$= -14 \qquad = -7 \qquad = 7$

$$C = \begin{bmatrix} 9 & 4 & -6 \\ 8 & 2 & -3 \\ -14 & -7 & 7 \end{bmatrix}$$

Adj B = transpose of C

$$= \begin{bmatrix} 9 & 8 & -14 \\ 4 & 2 & -7 \\ -6 & -3 & 7 \end{bmatrix}$$

68. (A) $\log_5 [\log_5\{\log_5(\log_5 x)\}] = 0$

$\log_5 [\log_5\{\log_5(\log_5 x)\}] = \log_5 1$

On comparing

$\log_5 \{\log_5(\log_5 x)\} = 1$

$\log_5 (\log_5 x) = 5^1$

$\log_5 x = 5^5$

$\log_5 x = 3125$

$x = 5^{3125}$

69.(C) a, b, c are in G.P.

$\log_e a, \log_e b, \log_e c$ are in A.P.

$1 + \log_e a,\ 1+ \log_e b,\ 1+ \log_e c$ are in A.P.

$\log_e ea, \log_e eb, \log_e ec$ are in A.P.

$\frac{1}{\log_e ea}, \frac{1}{\log_e eb}, \frac{1}{\log_e ec}$ are in H.P.

$\log_{ea} e, \log_{eb} e, \log_{ec} e$ are in H.P.

70. (C) Given that

$S_n = n^2 + 3n$

$S_{n-1} = (n-1)^2 + 3(n-1)$

$= n^2 + 1 - 2n + 3n - 3$

$= n^2 + n - 2$

$T_n = S_n - S_{n-1}$

$= (n^2 - 3n) - (n^2 + n - 2)$

$T_n = 2n + 2$

71. (D) $\tan^2 30° = \frac{1}{3}$, $\tan^2 45° = 1$, $\tan^2 60° = 3$

common ratio is same *i.e.* 3

$\tan^2 30°, \tan^2 45°, \tan^2 60°$ are in G.P.

72. (D) We know that

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n}{6}(n+1)(2n+1)$$

series $3 + 5 + 7 \ldots + (2n + 1)$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{n}{2}(2 \times 3 + (n-1)2)$$

$= n(3 + n - 1)$

$= n(n + 2)$

$$n^{th} \text{ term of series} = \frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{3 + 5 + 7 + \ldots + (2n+1)}$$

$$= \frac{\frac{n}{6}(n+1)(2n+1)}{n(n+2)}$$

$$= \frac{(n+1)(2n+1)}{6(n+2)}$$

73. (B) $x = \frac{y}{2} - \left(\frac{y}{2}\right)^2 + \left(\frac{y}{2}\right)^3 - \left(\frac{y}{2}\right)^4 + .. \infty$

$$x = \frac{\frac{y}{2}}{1 - \left(-\frac{y}{2}\right)} \qquad \left[\because S_\infty = \frac{a}{1-r}\right]$$

$$x = \frac{y}{2+y}$$

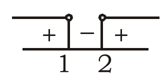
$2x + xy = y$

$2x = y(1 - x)$

$$y = \frac{2x}{1-x}$$

74. (D) $x^2 - 3x + 2 > 0$

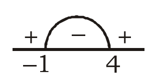
$(x - 2)(x - 1) > 0$



...(i)

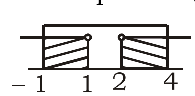
$x^2 - 3x - 4 \le 0$

$(x - 4)(x + 1) \le 0$



...(ii)

from equation (i) and (ii)



$-1 \le x < 1$ or $2 < x \le 4$

75. (C) $x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 1, 3$

$x = 1, 3$ is a factor of $x^2 - px^2 + q = 0$

then $x = 1, 3$ will satisfy the equation

$x^2 - px^2 + q = 0$ ...(i)

On putting the value of $x = 1$ in equation (i)

$(1)^2 - p(1)^2 + q = 0$

$p - q = 1$ ...(ii)

On putting the value of $x = 3$ in equation (i)

$(3)^2 - p \times (3)^2 + q = 0$

$9p - q = 9$ ...(iii)

On solving equation (ii) and equation (iii)

$p = 1$ and $q = 0$

So $(p, q) = (1, 0)$

76. (A) Equation $lx^2 - mx + m = 0$

roots = a, b

$$a + b = -\frac{(-m)}{l} = \frac{m}{l}$$

$$a.b = \frac{m}{l}$$

then $$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \frac{\alpha+\beta}{\sqrt{\alpha\beta}}$$

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \frac{\frac{m}{l}}{\sqrt{\frac{m}{l}}}$$

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \sqrt{\frac{m}{l}}$$

$$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} - \sqrt{\frac{m}{l}} = 0$$

77. (B) Let $y = \sqrt{5\sqrt{5\sqrt{5...\infty}}}$

$y = \sqrt{5y}$

$y^2 = 5y$

$y^2 - 5y = 0$

$y(y - 5) = 0$

$y = 0, 5$

78. Let $y = \sqrt{6+\sqrt{6+\sqrt{6+...\infty}}}$

$y = \sqrt{6+y}$

$y^2 = 6 + y$

$y^2 - y - 6 = 0$

$(y - 3)(y + 2) = 0$

$y = 3, -2$

79. (C) Let $z = \left[\frac{1-i}{1-(1+2i)^2}\right]$

$$z = \left[\frac{1-i}{1-(1+4i^2+4i)}\right]$$

$$z = \left[\frac{1-i}{1-(-3+4i)}\right]$$

$$z = \left[\frac{1-i}{4-4i}\right]$$

$$z = \frac{(1-i)}{4(1-i)}$$

$$z = \frac{1}{4}$$

Modulus of $z = |z| = \frac{1}{4}$

80.(A) $z = \frac{1-2i}{2+i} - \frac{2+i}{1-2i}$

$$z = \frac{(1-2i)(2-i)}{(2+i)(2-i)} - \frac{(2+i)(1+2i)}{(1-2i)(1+2i)}$$

$$z = \frac{2-4i-i+2i^2}{4-i^2} - \frac{2+4i+i+2i^2}{1-4i^2}$$

$$z = \frac{-5i}{5} - \frac{5i}{5}$$

$z = -i - i$

$z = -2i$

then

$z^2 + z\bar{z} = (-2i)^2 + (-2i)(2i)$

where $\bar{z}$ = conjugate of (z)

$= 4i^2 - 4i^2$

$= 0$

81.(B) $x + iy = \begin{vmatrix} 3i & 4i & 1 \\ 0 & 4 & -3i \\ 1 & -4i & 2 \end{vmatrix}$

$i \to -i$

$$x - iy = \begin{vmatrix} -3i & -4i & 1 \\ 0 & 4 & 3i \\ 1 & 4i & 2 \end{vmatrix}$$

$= -3i(8 - 12i^2) + 4i(0 - 3i) + 1(0 - 4)$

$= -3i(8 + 12) - 12i^2 - 4$

$= -60i + 12 - 4$

$= -60i + 8$

82. (D) Let $z = (2 - \omega + 2\omega^2)^{21}$

$= [2(1 + \omega^2) - \omega]^{21}$

$= [2(-\omega) - \omega]^{21}$ $[\because 1 + \omega + \omega^2 = 0]$

$= [-3\omega]^{21}$

$= -3^{21}\omega^{21}$

$= -3^{21}(\omega^3)^7$

$= -3^{21}$ $[\because \omega^3 = 1]$

83. (A) Let $z = \sqrt{i} + \sqrt{-i}$

We know that

$e^{i\theta} = \cos\theta + i\sin\theta$

at $\theta = \frac{\pi}{2}$

$e^{\frac{\pi i}{2}} = i$, $e^{-\frac{\pi i}{2}} = -i$

$z = e^{\frac{\pi i}{4}} + e^{-\frac{\pi i}{4}}$

$z = \cos\frac{\pi}{2} + i\sin\frac{\pi}{4} + \cos\left(\frac{\pi}{4}\right) - i\sin\frac{\pi}{4}$

$= 2\cos\frac{\pi}{4}$

$= 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$.

84. (A) A = {1, 4, 9, 16, 25, 36, 49, 64, 81}

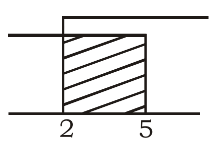
B = {2, 4, 6, 8, ...}

$A \cap B$ = {4, 16, 36, 64}

Number of elements = 4

85. (B) A = $\{(x, y)/\ x + y \le 5\}$

B = $\{(x, y)/\ x + y \ge 2\}$



$A \cap B = \{(x, y)/2 \le x + y \le 5\}$

Solution - (86-90)

Given that

n(U) = 46, only $n(H)$ = 20

only n(U) = 15, only $n(E)$ = 19

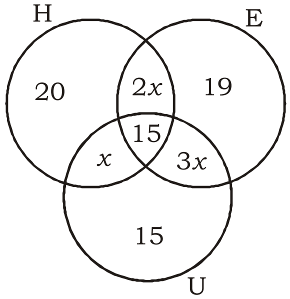
n(H ∩ E ∩ U) = 15

students who do not learn any language = 26

Let only $n(H \cap U) = x$

then only $n(H \cap E) = 2x$

only $n(E \cap U) = 3x$



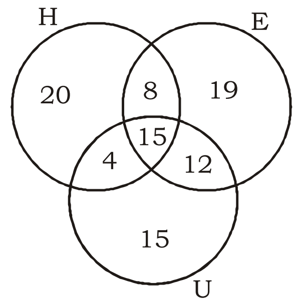
According to diagram

$n(U) = x + 15 + 3x + 15$

$46 = 4x + 30$

$4x = 16$

$x = 4$



86. (B) $n(H \cap E)$ = 8 + 15 = 23

87. (A) Students who learn atleast one language

= 20 + 8 +19 + 15 + 12 + 4 + 15

= 93

88. (B) Total number of students = 93 + 26

= 119

89. (C) Students who learn atleast two languages

= 15 + 4 + 8 + 12

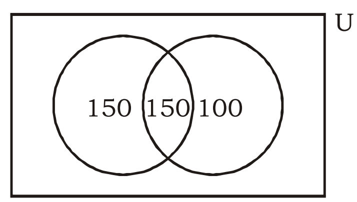
= 39

90. (A) Students who learn precisely two languages = 4 + 8 + 12

= 24

91. (D) ϕ = { } is correct.

92. (D)



n(U) = 600

n(A) = 300

n(B) = 250

$n(A \cap B)$ = 150

We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B)$ = 300 + 250 – 150

$n(A \cup B)$ = 400

$n(A' \cap B') = n(U) - n(A \cup B)$

= 600 – 400

= 200

93. (B) P = 3 sin θ + 4 cos θ.

We know that

$x = a\sin\theta + b\cos\theta$

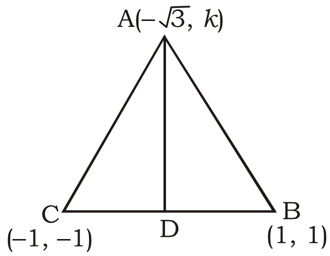
$-\sqrt{a^2 + b^2} \le x \le \sqrt{a^2 + b^2}$

then

$-\sqrt{(3)^2+(4)^2} \le P \le \sqrt{(3)^2+(4)^2}$

$-5 \le P \le 5$

Solution (94-96):



ΔABC is an equilateral triangle

AC = BC

$\sqrt{(-\sqrt{3}+1)^2+(k+1)^2} = \sqrt{(1+1)^2+(1+1)^2}$

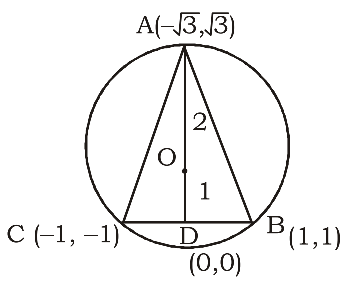
$3 + 1 - 2\sqrt{3} + k^2 + 1 + 2k = 4 + 4$

$k^2 + 2k - 2\sqrt{3} - 3 = 0$

On solving

$k = \sqrt{3}$

94. (B)



$k = \sqrt{3}$

point A = $(-\sqrt{3}\ ,\ \sqrt{3}\ )$

95. (C) Co-ordinate of point D = $\left(\frac{-1+1}{2}, \frac{-1+1}{2}\right)$

= (0, 0)

AD = $\sqrt{(\sqrt{3}-0)^2+(\sqrt{3}-0)^2}$

= $\sqrt{3+3}$

(Altitude) AD = $\sqrt{6}$

Radius of circumcircle (OA) = $\frac{2}{3}$ AD

= $\frac{2}{3} \times \sqrt{6}$

(R) = $\frac{2\sqrt{2}}{\sqrt{3}}$

96. (B) Altitude AD = $\sqrt{6}$

97. (B) (a, 0), (1, b) and (3, 2) are collinear.

Then

$$\begin{vmatrix} a & 0 & 1 \\ 1 & b & 1 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$a(b-2) + 0 + 1(2-3b) = 0$

$ab - 2a + 2 - 3b = 0$

$ab - 2a - 3b = -2$

98.(A) Equation $\frac{d^2y}{dx^2} + \cos x = 0$

On integrating both side

$\int \frac{d^2y}{dx^2}\,dx + \int \cos x\,dx = \int 0\,dx$

$\frac{dy}{dx} + \sin x = 0 + c$

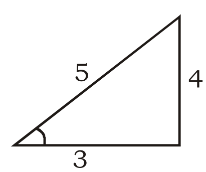
On integrating both side

$\int \frac{dy}{dx}\,dx + \int \sin x\,dx = \int c\,dx$

$y - \cos x = cx + d$

$y = \cos x + cx + d$

99. (A) $\cos^{-1}\frac{3}{5} + 2\cot^{-1} 3$



$= \tan^{-1}\frac{4}{3} + 2\tan^{-1}\frac{1}{3}$

$= \tan^{-1}\frac{4}{3} + \tan^{-1}\left[\frac{\frac{2}{3}}{1-\left(\frac{1}{3}\right)^2}\right]$

$\left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$= \tan^{-1}\frac{4}{3} + \tan^{-1}\left[\frac{3}{4}\right]$

$= \tan^{-1}\left[\frac{\frac{4}{3}+\frac{3}{4}}{1-\frac{4}{3}\times\frac{3}{4}}\right]$

$$= \tan^{-1}\left[\frac{\frac{8}{3}}{1-1}\right]$$

$$= \tan^{-1}(\infty) = \frac{\pi}{2}$$

100. (B)Given that

$f(x) = e^x$, $g(x) = \log x$

$gof(x) = g[f(x)]$

$= g[e^x]$

$= \log e^x$

$gof(x) = x$

differentiate both side w.r.t. 'x'

$gof'(x) = 1$

101. (C)

2	38	
2	19	0
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

$(38)_{10} = (100110)_2$

102. (C) $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k})$

$= (\hat{i} \cdot \hat{i})\vec{a} - (\hat{i} \cdot \vec{a})\hat{i} + (\hat{j} \cdot \hat{j})\vec{a} - (\hat{j} \cdot \vec{a})\hat{j} + (\hat{k} \cdot \hat{k})\vec{a} - (\hat{k} \cdot \vec{a})\hat{k}$

$[\because \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}]$

$= \vec{a} - (\hat{i} \cdot \vec{a})\hat{i} + \vec{a} - (\hat{j} \cdot \vec{a})\hat{j} + \vec{a} - (\hat{k} \cdot \vec{a})\hat{k}$

$= 3\vec{a} - \{(\hat{i} \cdot \vec{a})\hat{i} + (\hat{j} \cdot \vec{a})\hat{j} + (\hat{k} \cdot \vec{a})\hat{k}\}$

$= 3\vec{a} - \vec{a}$

$= 2\vec{a}$

103. (B) Vectors $(\hat{i} - 2\hat{j} + 3\hat{k})$ and $(3\hat{i} - 2\hat{j} - \hat{k})$

angle between two vectors

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{1 \times 3 + (-2) \times (-2) + 3 + (-1)}{\sqrt{(1)^2 + (-2)^2 + (3)^2}\sqrt{(3)^2 + (-2)^2 + (-1)^2}}$$

$$= \frac{3 + 4 - 3}{\sqrt{14}\sqrt{14}}$$

$$= \frac{4}{14}$$

$$\cos\theta = \frac{2}{7} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{7}\right)$$

104. (B)

Class	Frequency (f)	Commulative frequency (C)
0-10	5	5
10-20	6	11
20-30	12	23
30-40	**15**	**38**
40-50	18	56

$n = \Sigma f = 56$

$$\frac{n}{2} = \frac{56}{2} = 28$$

Hence medium class is (30 – 40).

$l_1 = 30$, $l_2 = 40$, $f = 15$, $C = 23$

$$\text{Median} = l_1 + \left(\frac{\frac{n}{2} - C}{f}\right) \times (l_2 - l_1)$$

$$= 30 + \frac{28 - 23}{15} \times (40 - 30)$$

$$= 30 + \frac{5}{15} \times 10$$

$$= 30 + 3.33 = 33.33$$

105. (D)

Class	x	frequency (f)	$f \times x$
0-10	5	5	25
10-20	15	6	150
20-30	25	12	300
30-40	35	15	525
40-50	45	18	810

$\Sigma f = 56$ $\quad \Sigma f \times x = 1810$

$$\text{Mean} = \frac{\Sigma f \times x}{\Sigma f}$$

$$= \frac{1810}{56}$$

$$= \frac{905}{28}$$

106. (B)

x_i	f_i	$f_i x_i$	$f_i x_i^2$
2	$\frac{1}{2}$	1	2
3	$\frac{1}{3}$	1	3
4	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{8}{3}$

$$\Sigma fi = 1,\ \Sigma f_i x_i = \frac{8}{3},\ \Sigma f_i x_i^2 = \frac{23}{3}$$

$$\text{variance } (\sigma^2) = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2$$

$$= \frac{\frac{23}{3}}{1} - \left(\frac{\frac{8}{3}}{1}\right)^2$$

$$= \frac{23}{3} - \frac{64}{9}$$

$$= \frac{5}{9}$$

107. Ellipse $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1, \alpha > \beta$

$$e^2 = 1 - \frac{\beta^2}{\alpha^2} \qquad \ldots(i)$$

and ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$

$$a^2 = 16, b^2 = 25$$

$$e^2 = 1 - \frac{a^2}{b^2}$$

$$e^2 = 1 - \frac{16}{25} \qquad \ldots(ii)$$

from equation (i) and equation (ii)

$$1 - \frac{\beta^2}{\alpha^2} = 1 - \frac{16}{25}$$

$$\frac{\beta^2}{\alpha^2} = \frac{16}{25}$$

$$\frac{\beta}{\alpha} = \frac{4}{5}$$

$$4\alpha = 5\beta$$

108. (B) Point (1, 1) and q(3, 4)

slope of line pq $m_1 = \frac{4-1}{3-1} = \frac{3}{2}$

line $3x + 4y = 1$

slope of line $m_2 = -\frac{3}{4}$

angle between lines

$$\tan q = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{3}{2} + \frac{3}{4}}{1 + \left(\frac{3}{2}\right)\left(-\frac{3}{4}\right)}\right|$$

$$\tan q = \left(\frac{\frac{9}{4}}{-\frac{1}{8}}\right)$$

$$\tan q = -18$$

$$q = \tan^{-1}(-18)$$

109. (A) $I = \int_{-2}^{-1} \frac{dx}{x^2 + 4x + 5}$

$$= \int_{-2}^{-1} \frac{dx}{(x+2)^2 + 1}$$

$$= \left[\tan^{-1}(x+2)\right]_{-2}^{-1}$$

$$= \tan^{-1}(1) - \tan^{-1} 0$$

$$= \frac{\pi}{4}$$

110. (D) $I = \int_{-\pi}^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

$$I = 2\int_0^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$$

$$\left[\because \int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x), & f(x) \text{ is even.} \\ 0, & f(x) \text{ is odd.} \end{cases}\right]$$

$$I = 2 \times \int_0^{2\times\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$$

$$I = 2 \times 2\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx \quad \ldots(i)$$

$$I = 4\int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{\sin^4 x + \cos^4 x} dx \quad \ldots(ii) \text{ Property IV}$$

on adding equation (i) and equation (ii)

$$2I = 4\int_0^{\frac{\pi}{2}} 1.dx$$

$$2I = 4[x]_0^{\frac{\pi}{2}}$$

$$2I = 4 \times \frac{\pi}{2}$$

$$I = \pi$$

111. (C) $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta.\sec^2\theta d\theta$

$$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sec\theta.\tan\theta\, d\theta$$

$$I = [\sec\theta]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$I = \sec\frac{\pi}{2} - \sec\frac{\pi}{4}$$

$$= \infty - \sqrt{2}$$

$$I = \infty$$

112. (B) $x^m y^n = 2(x + y)^{m+n}$

taking log both side

$m \log x + n \log y = \log 2 + (m + n) \log (x + y)$

differentiate both side w.r.t. 'x'

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = 0 + \frac{(m+n)}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\left[\frac{n}{y} - \frac{(m+n)}{x+y}\right]\frac{dy}{dx} = \frac{(m+n)}{x+y} - \frac{m}{x}$$

$$\frac{nx-my}{y}\frac{dy}{dx} = \frac{nx-my}{x}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

113. (A) $y = \cos^2 x^2$

differentiate both side w.r.t. 'x'

$$\frac{dy}{dx} = 2\cos x^2 (-\sin x^2) . (2x)$$

$$\frac{dy}{dx} = -2x \times 2\sin x^2 \cdot \cos x^2$$

$$\frac{dy}{dx} = -2x \sin 2x^2$$

again differentiate both side w.r.t. 'x'

$$\frac{d^2y}{dx^2} = -2[x\cos(2x^2)(2 \times 2x) + \sin 2x^2 \times 1]$$

$$\frac{d^2y}{dx^2} = -2[4x^2 \cos 2x^2 + \sin 2x^2]$$

$$= [-8x^2 \cos 2x^2 - 2\sin 2x^2]$$

114. (D) $f(x) = \begin{cases} \frac{2^x - 1}{\sqrt{2+x} - \sqrt{2}}, & -2 \le x < \infty, x \ne 0 \\ k, & x = 0 \end{cases}$

is continuous at everywhere

then $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0)$

$\lim_{x\to 0^+} f(x) = f(0)$

$$\lim_{x\to 0} \frac{2^x - 1}{\sqrt{2+x} - \sqrt{2}} = k$$

by L-Hospital's Rule

$$\lim_{x\to 0} \frac{2^x \log 2 - 0}{\frac{1}{2\sqrt{2+x}}} = k$$

$$\lim_{x\to 0} 2^x \log(2) \times [2\sqrt{2+x}] = k$$

$$2^o \log(2) \times \left(2\sqrt{2}\right) = k$$

$$k = 2\sqrt{2}\log 2$$

115. (A) $3f(x + 1) + f\left(\frac{1}{x+1}\right) = x$ ...(i)

On putting $x = 2$

$3f(3) + f\left(\frac{1}{3}\right) = 2$ ...(ii)

On putting $x = -\frac{2}{3}$ in equation (i)

$3f\left(\frac{1}{3}\right) + f(3) = \frac{-2}{3}$ ...(iii)

Equation (ii) × 3 – equation (iii)

$$9f(3) - f(3) = 6 + \frac{2}{3}$$

$$8f(3) = \frac{20}{3}$$

$$f(3) = \frac{5}{6}$$

116. (D) number of diagonals = $\frac{n(n-3)}{2}$ = 14

$n^2 - 3n = 28$

$n^2 - 3n - 28 = 0$

$(n-7)(n+4) = 0$

$n = 7, -4$

number of side = 7

117. (C) {0, 1, 2, 3, 4, 5}

5	5	4	3	2

= 5 × 5 × 4 × 3× 2
= 600

because '0' can not be put here.

The number of five digit numbers formed with given digits without any repetition of digits = 600.

118. (B) $\frac{(n+2)!+(n+1)(n-1)!}{(n+1)(n-1)!}$

$= \frac{(n+2)(n+1)n(n-1)!+(n+1)(n-1)!}{(n+1)(n-1)!}$

$= (n+2)n+1$

$= (n+1)^2$

= A perfect square number.

119. (B) $\frac{\sec(\pi+\theta).\sin\left(\frac{9\pi}{2}+\theta\right).\text{cosec}\left(\frac{11\pi}{2}-\theta\right)}{\sec(3\pi-\theta).\tan\left(\frac{3\pi}{2}+\theta\right).\cot\left(\frac{5\pi}{2}+\theta\right)}$

$= \frac{(-\sec\theta).\sin\left[2\times2\pi+\left(\frac{\pi}{2}+\theta\right)\right].\text{cosec}\left(3\times2\pi-\frac{\pi}{2}-\theta\right)}{\sec(2\pi+(\pi-\theta)).(-\cot\theta).\cot\left(2\pi+\frac{\pi}{2}+\theta\right)}$

$= \frac{(-\sec\theta).\sin\left(\frac{\pi}{2}+\theta\right).\text{cosec}\left[3\times2\pi-\left(\frac{\pi}{2}+\theta\right)\right]}{\sec(\pi-\theta).(-\cot\theta).\cot\left[2\pi+\left(\frac{\pi}{2}+\theta\right)\right]}$

$= \frac{(-\sec\theta).\cos\theta\left[-\text{cosec}\left(\frac{\pi}{2}+\theta\right)\right]}{(-\sec\theta).(-\cot\theta).\cot\left(\frac{\pi}{2}+\theta\right)}$

$= \frac{\cos\theta[-\sec\theta]}{(-\cot\theta)(-\tan\theta)}$

$= -1$

120. (A) $\frac{\sin 13^\circ+\cos 13^\circ}{\cos 13^\circ-\sin 13^\circ} - \frac{\cos 212^\circ}{\sin 212^\circ}$

$= \frac{1+\tan 13^\circ}{1-\tan 13^\circ} - \cot 212^\circ$

$= \tan(45^\circ + 13^\circ) - \cot(270^\circ - 58^\circ)$

$= \tan 58^\circ - \tan 58^\circ$

$= 0$

NDA (MATHS) MOCK TEST - 72 (Answer Key)

1.	(C)	21.	(C)	41.	(C)	61.	(C)	81.	(B)	101.	(C)
2.	(B)	22.	(C)	42.	(C)	62.	(D)	82.	(D)	102.	(C)
3.	(A)	23.	(D)	43.	(B)	63.	(B)	83.	(A)	103.	(B)
4.	(C)	24.	(C)	44.	(D)	64.	(B)	84.	(A)	104.	(B)
5.	(D)	25.	(D)	45.	(D)	65.	(C)	85.	(B)	105.	(D)
6.	(C)	26.	(B)	46.	(B)	66.	(A)	86.	(B)	106.	(B)
7.	(C)	27.	(C)	47.	(A)	67.	(D)	87.	(A)	107.	(A)
8.	(B)	28.	(A)	48.	(C)	68.	(A)	88.	(B)	108.	(B)
9.	(A)	29.	(B)	49.	(D)	69.	(C)	89.	(C)	109.	(A)
10.	(A)	30.	(B)	50.	(A)	70.	(C)	90.	(A)	110.	(D)
11.	(D)	31.	(B)	51.	(C)	71.	(D)	91.	(D)	111.	(C)
12.	(C)	32.	(B)	52.	(B)	72.	(B)	92.	(D)	112.	(B)
13.	(A)	33.	(C)	53.	(C)	73.	(B)	93.	(B)	113.	(A)
14.	(D)	34.	(A)	54.	(D)	74.	(D)	94.	(B)	114.	(D)
15.	(C)	35.	(D)	55.	(A)	75.	(C)	95.	(C)	115.	(A)
16.	(A)	36.	(A)	56.	(B)	76.	(A)	96.	(B)	116.	(D)
17.	(B)	37.	(B)	57.	(B)	77.	(B)	97.	(B)	117.	(C)
18.	(C)	38.	(A)	58.	(C)	78.	(C)	98.	(A)	118.	(C)
19.	(D)	39.	(D)	59.	(D)	79.	(C)	99.	(A)	119.	(B)
20.	(A)	40.	(D)	60.	(B)	80.	(A)	100.	(B)	120.	(A)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*