NDA MATHS MOCK TEST - 78 (SOLUTION)

1. (C) $\left(4x^3 + \frac{1}{2x}\right)^7$

general term in the expansion

$T_{r+1} = {}^7C_r\,(4x^3)^{7-r}\left(\frac{1}{2x}\right)^r$

$= {}^7C_r\,4^{7-r}\left(\frac{1}{2}\right)^r x^{21-4r}$

$21 - 4r = 5$

$r = 4$

coefficient of x^5 in the expansion of $\left(4x^3 + \frac{1}{2x}\right)^7$

$\Rightarrow {}^7C_4\,4^3\left(\frac{1}{2}\right)^4$

$\Rightarrow \frac{7!}{4!3!} \times \frac{2^6}{2^4}$

$\Rightarrow 140$

2. (B) digits {1, 3, 0, 3, 4, 4}

Numbers greater than 1000000 by using the given digits $= \frac{6!}{2!2!}$

$= 180$

but numbers starting with '0' are not greater than 100000.

numbers starting with '0' $= \frac{5!}{2!2!}$

$= 30$

Hence total numbers greater than 100000 are = 180 – 30

= 150

3. (A) Physics paper consists of 12 questions.

Total no. of ways $= {}^6C_3 \times {}^6C_5 + {}^6C_4 \times {}^6C_4 + {}^6C_5 \times {}^6C_3$

$\Rightarrow 20 \times 6 + 15 \times 15 + 6 \times 20$

$\Rightarrow 120 + 225 + 120$

$\Rightarrow 465$

4. (A) Given $\left[\sqrt[3]{3} + \frac{1}{\sqrt[4]{4}}\right]^n$

fifth term from the beginning

$T_5 = T_{4+1} = {}^nC_4\left(3^{\frac{1}{3}}\right)^{n-4}\left(\frac{1}{2^{\frac{1}{2}}}\right)^4$

$= {}^nC_4\,3^{\frac{n-4}{3}} \times 2^{-2}$

fifth term from the end

$T_{n+1-4} = T_{(n-4)+1} = {}^nC_{n-4}\left(3^{\frac{1}{3}}\right)^4\left(\frac{1}{2^{\frac{1}{2}}}\right)^{n-4}$

$= {}^nC_4\,3^{\frac{4}{3}}\,2^{-\frac{n-4}{2}}$

ratio $\frac{{}^nC_4\,3^{\frac{n-4}{3}}.2^{-2}}{{}^nC_4\,3^{\frac{4}{3}}.2^{-\frac{n-4}{2}}} = \frac{6\sqrt{2}}{1}$

$3^{\frac{n-8}{3}} \cdot 2^{\frac{n-8}{2}} = \frac{6\sqrt{2}}{1}$

$3^{\frac{n-8}{3}} . 2^{\frac{n-8}{2}} = 3 \times 2^{\frac{3}{2}}$

on comparing

$\frac{n-8}{3} = 1$

$n = 11$

5. (C) $\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8}$

$\Rightarrow \frac{1}{2}\left[2\sin^2\frac{\pi}{8} + 2\sin^2\frac{3\pi}{8} + 2\sin^2\frac{5\pi}{8} + 2\sin^2\frac{7\pi}{8}\right]$

$\Rightarrow \frac{1}{2}\left[1 - \cos\frac{\pi}{4} + 1 - \cos\frac{3\pi}{4} + 1 - \cos\frac{5\pi}{4} + 1 - \cos\frac{7\pi}{4}\right]$

$\Rightarrow \frac{1}{2}\left[1 - \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}}\right]$

$\Rightarrow \frac{1}{2} \times 4 = 2$

6. (B) $\frac{\cos 5x - 2\cos 3x + \cos x}{\sin 5x - \sin x}$

$\Rightarrow \frac{(\cos 5x + \cos x) - 2\cos 3x}{\sin 5x - \sin x}$

$\Rightarrow \frac{2\cos 3x.\cos 2x - 2\cos 3x}{2\cos 3x.\sin 2x}$

$\Rightarrow \frac{2\cos 3x(\cos 2x - 1)}{2\cos 3x\sin 2x}$

$\Rightarrow \frac{-2\sin^2 x}{2\sin x.\cos x}$

$\Rightarrow -\tan x$

7. (B) We know that

$$\cos A \cos(60 - A) \cos(60 + A) = \frac{1}{4} \cos 3A$$

8. (A) We know that

$$-\sqrt{a^2 + b^2} \le a \sin\theta + b\cos\theta \le \sqrt{a^2 + b^2}$$

then $-\sqrt{4^2 + 3^2} \le 4\sin\theta + 3\cos\theta \le \sqrt{4^2 + 3^2}$

$-5 \le 4\sin\theta + 3\cos\theta \le 5$

$-5 + 5 \le 4\sin\theta + 3\cos\theta + 5 \le 5 + 5$

$0 \le 4\sin\theta + 3\cos\theta + 5 \le 10$

minimum value of $(4\sin\theta + 3\cos\theta + 5) = 0$

9. (D) $\begin{vmatrix} 2x-1 & 2x-1 & x^2+2x+3 \\ 3x-3 & 3x & 2x^2+3x+3 \\ x-2 & x+1 & x^2+x \end{vmatrix}$

$R_2 \to R_2 - R_1$

$$\Rightarrow \begin{vmatrix} 2x-1 & 2x-1 & x^2+2x+3 \\ x-2 & x+1 & 2x^2+x \\ x-2 & x+1 & x^2+x \end{vmatrix}$$

$= 0$ [$\because$ Two rows are identical.]

10.(B) The given equations have infinite many solution.

then $\frac{3}{a} = \frac{4}{-(a-b)} = \frac{9}{18}$

$\frac{3}{a} = \frac{9}{18}$

$a = 6$

and $\frac{4}{-(a-b)} = \frac{9}{18}$

$-a + b = 8$

$-6 + b = 8$

$b = 14$

Hence $7a = 3b$

11. (C) Let $a - ib = \sqrt{4 - 3i}$

on squaring

$(a^2 - b^2) - (2ab)i = 4 - 3i$

on comparing

$a^2 - b^2 = 4$ and $2ab = 3$ ...(i)

we know that

$(a^2 + b^2)^2 - 4a^2b^2 = (a^2 - b^2)^2$

$(a^2 + b^2)^2 - 9 = 16$

$a^2 + b^2 = 5$ ...(ii)

from equation (i) and equation (ii)

$a^2 = \frac{9}{2}$ and $b^2 = \frac{1}{2}$

$a = \pm\frac{3}{\sqrt{2}}$ $b = \pm\frac{1}{\sqrt{2}}$

square root of $(4 - 3i) = \pm\left(\frac{3-i}{\sqrt{2}}\right)$

12. (C) $\frac{3x}{9x^2 + 24x + 7} < \frac{1}{3x+2}$

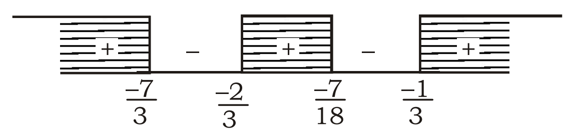
$$\frac{3x}{(3x+7)(3x+1)} - \frac{1}{(3x+2)} < 0$$

$$\frac{-18x-7}{(3x+7)(3x+2)(3x+1)} < 0$$

$$\frac{18x+7}{(3x+7)(3x+2)(3x+1)} > 0$$

equating each factor equal to '0'.

$x = \frac{-7}{18}, \frac{-7}{3}, \frac{-2}{3}, \frac{-1}{3}$



$$x \in \left(-\infty, \frac{-7}{3}\right) \cup \left(\frac{-2}{3}, \frac{-7}{18}\right) \cup \left(-\frac{1}{3}, \infty\right)$$

13. (C) $\lim_{x\to\frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$$\Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{\cos x - 0}{1}$$

$$\Rightarrow \frac{\cos\frac{\pi}{2}}{1}$$

$\Rightarrow 0$

14. (B) $\lim_{x\to 2} \frac{x+2}{x^2 - 5x + 4}$

$$\Rightarrow \frac{2+2}{(2)^2 - 5\times 2 + 4}$$

$$\Rightarrow \frac{4}{-2} = -2$$

15. (A) $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

Let $x^2 = \cos 2\theta \Rightarrow 2\theta = \cos^{-1}x^2$

$$\Rightarrow \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right]$$

$$\Rightarrow \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]$$

$$= \frac{\pi}{4} - \theta$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x^2$$

16. (C) $f(x) = \dfrac{\sqrt{4x^2-9}}{x-6}$

$4x^2 - 9 \geq 0$ $\quad\quad$ $x - 6 \neq 0$

$4x^2 \geq 9$ $\quad\quad$ $x \neq 6$

$$x^2 \geq \frac{9}{4}$$

$$x \geq \pm\frac{3}{2}$$

$$x \in \left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right) - \{6\}$$

17. (A) sin 60 + sin 120 + sin 240 + sin 300

$\Rightarrow$ sin 60 + sin (90 + 30) + sin (270 − 30) + sin(360 − 60)

$\Rightarrow$ sin 60 + cos 30 – cos 30 – sin 60

$\Rightarrow 0$

18. (C) $\lim\limits_{x\to 0}\dfrac{\sin(5+2x)-\sin(5-2x)}{\sin x}$ $\left[\dfrac{0}{0}\right]$ form

by L-Hospital's Rule

$$\Rightarrow \lim_{x\to 0}\frac{2\cos(5+2x)+2\cos(5-2x)}{\cos x}$$

$$\Rightarrow \frac{2\cos 5 + 2\cos 5}{\cos 0}$$

$\Rightarrow 4\cos 5$

19. (A) Series $\dfrac{1}{1\times 2} + \dfrac{1}{2\times 3} + \dfrac{1}{3\times 4} + \dfrac{1}{4\times 5} + \ldots$

$$\frac{1}{n\times(n+1)}$$

$$\Rightarrow \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$\Rightarrow 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1}$$

$$\Rightarrow 1 - \frac{1}{n+1}$$

$$\Rightarrow \frac{n}{n+1}$$

20. (C) Hyperbola

$3x^2 - 4y^2 = 5$

$$\frac{x^2}{\frac{5}{3}} - \frac{y^2}{\frac{5}{4}} = 1$$

$$a^2 = \frac{5}{3},\ b^2 = \frac{5}{4}$$

$$a = \frac{\sqrt{5}}{\sqrt{3}},\ b = \frac{\sqrt{5}}{2}$$

eccentricity $e^2 = 1 + \dfrac{b^2}{a^2}$

$$e^2 = 1 + \frac{3}{4}$$

$$e = \frac{\sqrt{7}}{2}$$

$$\text{foci} = (\pm ae, 0) = \left(\pm\frac{\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{7}}{2}, 0\right)$$

$$= \left(\pm\frac{1}{2}\sqrt{\frac{35}{3}}, 0\right)$$

21. (D) **Reflexive**

aRa

since 'a' can not be a mother of 'a'.

which is not possible.

Symmetric

$aRb \Leftrightarrow bRa$

which is not possible.

Transitive

aRb, bRc then aRc

which is not possible.

22. (B) **Statement (I)**

We know that

$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n}{6}(n+1)(2n+1)$

for first 15 natural numbers

$1^2 + 2^2 + 3^2 + ... + 15^2 = \frac{15}{6} \times 16 \times 31$

$= 1240$

Statement (I) is incorrect.

Statement (II)

We know that

$1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{1}{2}n(n+1)\right]^2$

for first 15 natural numbers

$1^3 + 2^3 + 3^3 + ... + 15^3 = \left[\frac{1}{2} \times 15 \times 16\right]^2$

$= 14400$

Statement (II) is correct.

23. (B) $\sum_{n=0}^{7}\left(i^n - i^{n+1}\right)$

$\Rightarrow (i^0 - i^1) + (i^1 - i^2) + (i^2 - i^3) + (i^3 - i^4) + (i^4 - i^5) + (i^5 - i^6) + (i^6 - i^7) + (i^7 - i^8)$

$\Rightarrow 1 - i^8$

$\Rightarrow 1 - 1 = 0$

24. (C) Given that $A = \begin{bmatrix} 2 & -1 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

$(A^T) = B = \begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 5 & 4 \end{bmatrix}$

B is the transpose of A.

25. (A) Matrix $\begin{bmatrix} 1 & 2 & -4 \\ 2 & x & -3 \\ -4 & 0 & 0 \end{bmatrix}$ is a singular,

if $\begin{vmatrix} 1 & 2 & -4 \\ 2 & x & -3 \\ -4 & 0 & 1 \end{vmatrix} = 0$

$1(x - 0) - 2(2 - 12) - 4(0 + 4x) = 0$

$15x = 20$

$x = \frac{4}{3}$

26. (C) Given that $n = 55$

number of diagonals $= \frac{n(n-3)}{2}$

$= \frac{55 \times 52}{2}$

$= 55 \times 26$

$= 1430$

27. (D) $n(S) = {}^7C_3$ [1, 2, 3, 4, 5, 6, 7]

two digits are chosen from (2, 4, 6) and 1 digit is chosen [1, 3, 5, 7]

$n(E) = {}^3C_2 \times {}^4C_1$

$P(E) = \frac{n(E)}{n(S)} = \frac{{}^3C_2 \times {}^4C_1}{{}^7C_3}$

$= \frac{12}{35}$

28. (A) Let $A = \begin{vmatrix} 1 & 2 & 3 \\ -2 & 0 & 5 \\ 3 & 1 & -4 \end{vmatrix}$

co-factors of A

$C_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 5 \\ 1 & -4 \end{vmatrix}, C_{12} = (-1)^{1+2}\begin{vmatrix} -2 & 5 \\ 3 & -4 \end{vmatrix}, C_{13} = (-1)^{1+3}\begin{vmatrix} -2 & 0 \\ 3 & 1 \end{vmatrix}$

$= -5 \qquad = 7 \qquad = -2$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix}, C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 3 & -4 \end{vmatrix}, C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$

$= 11 \qquad = -13 \qquad = 5$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}, C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ -2 & 5 \end{vmatrix}, C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ -2 & 0 \end{vmatrix}$

$= 10 \qquad = -11 \qquad = 4$

$C = \begin{vmatrix} -5 & 7 & -2 \\ 11 & -13 & 5 \\ 10 & -11 & 4 \end{vmatrix}$

Adj A = transpose of C

$= \begin{vmatrix} -5 & 11 & 10 \\ 7 & -13 & -11 \\ -2 & 5 & 4 \end{vmatrix}$

29. (B) $R = \{(1, 1), (3, 3), (4, 4)\}$

1R1, 3R3, 4R4

So R is a reflexive relation.

30. (B) Given that $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & -1 & 0 \\ 2 & 0 & -3 \end{bmatrix}$

$$3X + 2A = 0$$
$$3X = -2A$$
$$X = -\frac{2}{3}A$$
$$X = -\frac{2}{3}\begin{bmatrix} 3 & 2 & -1 \\ 1 & -1 & 0 \\ 2 & 0 & -3 \end{bmatrix}$$
$$X = \begin{bmatrix} -2 & -\frac{4}{3} & \frac{2}{3} \\ \frac{-2}{3} & \frac{2}{3} & 0 \\ \frac{-4}{3} & 0 & 2 \end{bmatrix}$$

31. (C) $\Rightarrow \cos^2 10 + \cos^2 20 + \cos^2 30 + \cos^2 40 + \cos^2 50 + \cos^2 60 + \cos^2 70 + \cos^2 80 + \cos^2 90$

$\Rightarrow \cos^2 10 + \cos^2 20 + \cos^2 30 + \cos^2 40 + \sin^2 40 + \sin^2 30 + \sin^2 20 + \sin^2 10 + 0$

$\Rightarrow \cos^2 10 + \sin^2 10 + \cos^2 20 + \sin^2 20 + \cos^2 30 + \sin^2 30 + \cos^2 40 + \sin^2 40$

$\Rightarrow 1 + 1 + 1 + 1$

$\Rightarrow 4$

32. (A) Given numbers (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

4 digit odd numbers when even numbers (2, 4, 6, 8) put in the first place and odd numbers (1, 3, 5, 7, 9) put in the last place

4	8	7	5

$= 4 \times 8 \times 7 \times 5 = 1120$

'0' can not put here

4 digit odd number when odd numbers (1, 3, 5, 7, 9) put in the first place and last place

5	8	7	4

$= 5 \times 8 \times 7 \times 4 = 1120$

total number = 1120 + 1120
= 2240

33. (B) Series $\sqrt{3} + \sqrt{12} + \sqrt{48} + \sqrt{192} + \ldots$

$\Rightarrow \sqrt{3} + 2\sqrt{3} + 4\sqrt{3} + 8\sqrt{3} \ldots$

$\Rightarrow \sqrt{3}\ (1 + 2 + 4 + 8 + \ldots)$

$$\Rightarrow \sqrt{3}\left[\frac{1\times(2^5 - 1)}{2-1}\right] \quad \left[\because S_n = \frac{a(r^n - 1)}{r-1} \text{ for G.P.}\right]$$

$$\Rightarrow \sqrt{3}\ \frac{(32-1)}{1}$$

$= 3\sqrt{3}$

34. (C) We know that

43 < 45

cos 43 > cos 45 and sin 43 < sin 45

sin 43 < cos 45

then cos 43 > cos 45 > sin 43

So cos 43 – sin 43 > 0 (positive)

Statement (I) is correct.

(II) 48 > 45

cos 48 < cos 45 and sin 48 > sin 45

sin 48 > cos 45

then cos 48 < cos 45 < sin 48

cos 48 – sin 48 < 0 (negative)

Statement (II) is correct.

35. (A) $I = \int_a^b \frac{x^3 - \tan x}{\sec x}\,dx$

where $a + b = 0$

$b = -a$

$$I = \int_a^{-a} \frac{x^3 - \tan x}{\sec x}\,dx$$

$$I = 0 \left[\because \int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is even} \\ 0\ , & \text{if } f(x) \text{ is odd} \end{cases}\right]$$

36. (A) $f(x) = 3x^2 - \log|x|$

$$f'(x) = 6x - \frac{1}{x}$$

$$f'(x) = \frac{6x^2 - 1}{x}$$

(i) $6x^2 - 1 \geq 0$ and $x > 0$

$$x^2 \geq \frac{1}{6}$$

$$x \geq \pm\frac{1}{\sqrt{6}}$$

$$x \geq \frac{1}{\sqrt{6}},\ x \leq \frac{-1}{\sqrt{6}} \text{ and } x > 0$$

then $\boldsymbol{x \geq \frac{1}{\sqrt{6}}}$

(ii) $6x^2 - 1 \leq 0$ and $x < 0$

$x \leq \pm \frac{1}{\sqrt{6}}$

$x \leq \frac{1}{\sqrt{6}}$ and $x \geq -\frac{1}{\sqrt{6}}$ and $x < 0$

$x \in \left[\frac{-1}{\sqrt{6}}, 0\right)$

Hence $\frac{-1}{\sqrt{6}} \leq x < 0$ or $x \geq \frac{1}{\sqrt{6}}$

37. (C) Let $y = x - \sin x$ and $z = \log(x - \cos x)$

$\frac{dy}{dx} = 1 - \cos x$, $\frac{dz}{dx} = \frac{1}{x - \cos x}(1 + \sin x)$

then $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$= \frac{(1 - \cos x)(x - \cos x)}{(1 + \sin x)}$

38. (B) $\frac{x^a}{y^b} = (x + y)^{a-b}$

taking log both side

$a \log x - b \log y = (a - b) \log (x + y)$

on differentiating both side w.r.t 'x'

$\frac{a}{x} - \frac{b}{y}\frac{dy}{dx} = (a - b) \times \frac{1}{x + y}\left[1 + \frac{dy}{dx}\right]$

$\frac{a}{x} - \frac{a - b}{x + y} = \left[\frac{a - b}{x + y} + \frac{b}{y}\right] \frac{dy}{dx}$

$\frac{ax + ay - ax + bx}{x(x + y)} = \left[\frac{ay - by + bx + by}{y(x + y)}\right]\frac{dy}{dx}$

$\frac{ay + bx}{x} = \frac{ay + bx}{y}\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y}{x}$

$\frac{dy}{dx} - \frac{y}{x} = 0$

39. (B) Given that

$f'(x) = \frac{x^2}{3} + 2Kx + 5$

on integrating

$f(x) = \frac{1}{3}\frac{x^3}{3} + 2K.\frac{x^2}{2} + 5x + c$

$f(x) = \frac{x^3}{9} + Kx^2 + 5x + c$ ...(i)

$f(0) = 0$

then $c = 0$

from equation (i)

$f(x) = \frac{x^3}{9} + Kx^2 + 5x$

$f(1) = -1$

$-1 = \frac{1}{9} + K + 5$

$K = \frac{-55}{9}$

40. (C) Given that

$f'(x) = \frac{x^2}{3} + 2Kx + 5$

$K = -\frac{55}{9}$

$f'(x) = \frac{x^2}{3} - \frac{110}{9}x + 5$

on differentiating both side

$f''(x) = \frac{2x}{3} - \frac{110}{9}$

$x = 6$

$f''(6) = \frac{2 \times 6}{3} - \frac{110}{9}$

$= \frac{36 - 110}{9}$

$= -\frac{74}{9}$

41. (C) Given that A = {1, 3}, B = {2, 4} and C = {1, 4}

$(A \times B)$ = {(1, 2), (1, 4), (3, 2), (3, 4)} and

$(A \times C)$ = {(1, 1), (1, 4), (3, 1), (3, 4)}

$(A \times B) \cap (A \times C)$ = {(1, 4), (3, 4)}

cardinality of $(A \times B) \cap (A \times C) = 2$

42. (B)

43. (D)

44. (B) Equation

$x^2 - 4|x| + 3 = 0$

$|x|^2 - 3|x| - |x| + 3 = 0$

$(|x| - 3)(|x| - 1) = 0$

$|x| - 3 = 0$ $|x| - 1 = 0$

$|x| = 3$ $|x| = 1$

$x = \pm 3$ $x = \pm 1$

four roots of equation are possible.

45. (B) We know that

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$$

$$1 + \sum_{r=1}^{n} C(n, r) = 2^n$$

$$\sum_{r=1}^{n} C(n, r) = 2^n - 1$$

46. (C)

2	79	
2	39	1
2	19	1
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

$(79)_{10} = (1001111)_2$

47. (B) Given that

$A = \{1, 2, 3, 4, 5\}$ and $B = \{a, b, c\}$

no. of element in A = 5 and no. of element in B = 3

then

no. of element in AB

= (no. of element in A) × (no. of element in B)

= 5 × 3

= 15

48. (A) Given that

$$\begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix} \times \begin{bmatrix} -3 & -2 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 9 & -7 \\ 37 & k \end{bmatrix}$$

$$\begin{bmatrix} -6+15 & -4-3 \\ 12+25 & 8-5 \end{bmatrix} = \begin{bmatrix} 9 & -7 \\ 37 & k \end{bmatrix}$$

$$\begin{bmatrix} 9 & -7 \\ 37 & 3 \end{bmatrix} = \begin{bmatrix} 9 & -7 \\ 37 & k \end{bmatrix}$$

on comparing

$k = 3$

49. (C) time = 7 : 30

angle between minute hand and hour hand

$$\theta = \left|\frac{11M - 60H}{2}\right|$$

$$= \left|\frac{11 \times 30 - 60 \times 7}{2}\right|$$

$= 45°$

$$= \left(\frac{\pi}{4}\right)^c$$

50. (B) $\frac{dy}{dx} + y = \left(1 - x\frac{dy}{dx}\right)^{-3}$

$$\left(\frac{dy}{dx} + y\right) = \frac{1}{\left(1 - x\frac{dy}{dx}\right)^3}$$

$$\left(\frac{dy}{dx} + y\right)\left[1 - x^3\left(\frac{dy}{dx}\right)^3 - 3x\frac{dy}{dx} + 3x^2\left(\frac{dy}{dx}\right)^2\right] = 1$$

$$-x^3\left(\frac{dy}{dx}\right)^4 + (3x^2 - x^3y)\left(\frac{dy}{dx}\right)^3 + 3(x^2y - x)\left(\frac{dy}{dx}\right)^2 - 3xy\frac{dy}{dx} = 1$$

degree = 4

51. (D) $\frac{dy}{dx} = 1 + x^2 - y^2 - x^2y^2$

$$\frac{dy}{dx} = 1(1 + x^2) - y^2(1 + x^2)$$

$$\frac{dy}{dx} = (1 + x^2)(1 - y^2)$$

$$\frac{dy}{1^2 - y^2} = (1 + x^2)dx$$

on integrating

$$\frac{1}{2 \times 1}\left[\log\left|\frac{1+y}{1-y}\right|\right] = x + \frac{x^3}{3} + c$$

$$\left[\because \int \frac{1}{a^2 - x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c\right]$$

$$\log\left|\frac{1+y}{1-y}\right| = 2x + \frac{2x^3}{3} + c$$

52. (C) Let the centre of circle = $(0, k)$

equation of circle

$(x - 0)^2 + (y - k)^2 = r^2$

$x^2 + (y - k)^2 = r^2$ ... (i)

passing through the point (0, 0)

$0 + k^2 = r^2$

$k = r$

from equation (i)

$x^2 + (y - k)^2 = k^2$

$x^2 + y^2 = 2yk$ ...(ii)

differential both *w.r.t.* 'x'

$$2x + 2y\frac{dy}{dx} = 2k\frac{dy}{dx}$$

$$2xy + 2y^2\frac{dy}{dx} = \frac{dy}{dx}(2yk)$$

$$2xy + 2y^2\frac{dy}{dx} = \frac{dy}{dx}(x^2 + y^2)$$

[from equation (ii)]

$$(x^2 + y^2 - 2y^2)\frac{dy}{dx} = 2xy$$

$$(x^2 - y^2)\frac{dy}{dx} = 2xy$$

53. (C) Sample space $n(S) = 2^4 = 16$

more than 2 times tail

$n(E) = {}^4C_3 + {}^4C_4$

$= 4 + 1 = 5$

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{16}$$

54. (B) Given that

$$P(\text{not A}) = 0.6,\ P(A \cup B) = 0.4,\ P\left(\frac{B}{A}\right) = 0.2$$

$P(A) = 1 - P(\text{not A})$

$= 1 - 0.6$

$P(A) = 0.4$

we know that

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$0.2 = \frac{P(A \cap B)}{0.4}$$

$P(A \cap B) = 0.08$

we know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.4 = 0.4 + P(B) - 0.08$

$P(B) = 0.08$

55. (C) Given that

$$2ae = 25 \qquad \text{and } e = \frac{1}{\sqrt{2}}$$

$$2a \times \frac{1}{\sqrt{2}} = 25$$

$$a = \frac{25\sqrt{2}}{2}$$

$$\text{then } e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{1}{2} = 1 - \frac{b^2 \times 4}{625 \times 2}$$

$$\frac{2b^2}{625} = \frac{1}{2}$$

$$b^2 = \frac{625}{4}$$

$$b = \frac{25}{2}$$

length of minor axis $= 2b$

$$= 2 \times \frac{25}{2} = 25$$

56. (A)

57. (C) A box contains 5 white and 3 black balls two balls are drawn at random one after the other.

$$\text{required probability} = \frac{5}{8} \times \frac{4}{7}$$

$$= \frac{5}{14}$$

58. (C) The correlation coefficient between x and y

$$r = \sqrt{b_{yx} \times b_{xy}}$$

$$r = \sqrt{\frac{-3}{16} \times \frac{-4}{3}}$$

$$r = -\frac{1}{2}$$

59. (A) Given that

$$\left|\frac{Z - 3i}{3i + Z}\right| = 1$$

Let $Z = x + iy$

$$\left|\frac{x + i(y-3)}{x + (y+3)i}\right| = 1$$

$$|x + i(y - 3)| = |x + (y + 3)i|$$

$$\sqrt{x^2 + (y-3)^2} = \sqrt{x^2 + (y+3)^2}$$

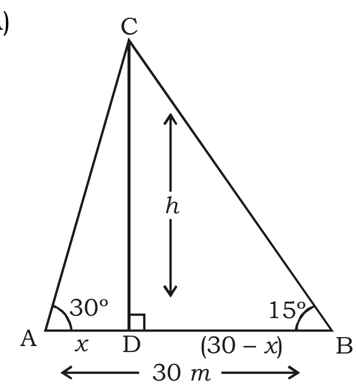
$$x^2 + y^2 + 9 - 6y = x^2 + y^2 + 9 + 6y$$

$$12y = 0$$

$$y = 0$$

locus of Z is a line.

60. (A)



Let height of aeroplane above the ground (CD) = h m
and AD = x m
In ΔACD

$$\tan 30 = \frac{CD}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = h\sqrt{3}$$

In ΔBCD

$$\tan 15 = \frac{CD}{BD}$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h}{30-x}$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h}{30-h\sqrt{3}}$$

$$30(\sqrt{3} - 1) = 4h$$

$$h = \frac{15\left(\sqrt{3}-1\right)}{2}$$

$$h = \frac{15}{\sqrt{3}+1}$$

61. (D) Let $f(x) = \dfrac{\sin x}{\sqrt{1-\cos x}}$

L.H.L. = $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} f(0 - h)$

$$= \lim_{h\to 0} \frac{\sin(0-h)}{\sqrt{1-\cos(0-h)}}$$

$$= \lim_{h\to 0} \frac{-\sin h}{\sqrt{1-\cos h}}$$

$$= \lim_{h\to 0} \frac{-2\sin\frac{h}{2}.\cos\frac{h}{2}}{\sqrt{2}\sin\frac{h}{2}} = -\sqrt{2} \times 1$$

$$= -\sqrt{2}$$

R.H.L. $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0} f(0 + h)$

$$= \lim_{h\to 0} \frac{\sin(0+h)}{\sqrt{1-\cos(0+h)}}$$

$$= \lim_{h\to 0} \frac{\sin h}{\sqrt{1-\cos h}}$$

$$= \lim_{h\to 0} \frac{2\sin\frac{h}{2}.\cos\frac{h}{2}}{\sqrt{2}\sin\frac{h}{2}}$$

$$= \sqrt{2} \times 1$$

$$= \sqrt{2}$$

L.H.L. ≠ R.H.L.
limit does not exist.

62. (C)

```
 11001
 10110
101111
```

then $(11001)_2 + (10110)_2 = (101111)_2$

63. (C) Let $z = \dfrac{(1-i)(2+i)}{(i-3)}$

$$z = \frac{(3-i)}{(i-3)}$$

$$z = -1$$

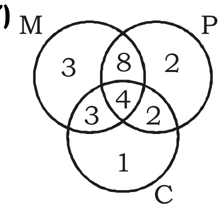
argument (θ) = $\tan^{-1} \dfrac{b}{a}$

$$= \tan^{-1}\left(\frac{0}{-1}\right)$$

$$= 0$$

64. (B)

(65-67)



Given that
total students = 35
$n(M) = 18$, $n(P) = 16$, $n(C) = 10$
$n(M \cap C) = 7$, $n(M \cap P) = 12$, $n(P \cap C) = 6$
$n(M \cap C \cap P) = 4$,
according to diagram
only $n(M) = 3$, only $n(P) = 2$
only $n(C) = 1$

65. (A) The number of students who had taken only chemistry = 1.
66. (C) The number of students who had taken only two subjects.
⇒ 8 + 3 + 2 = 13
67. (B) The student who had not taken any subjects
= 35 – (8 + 3 + 2 + 4 + 3 + 2 + 1)
= 12

68. (B) We know that

$$\cos 36° = \frac{\sqrt{5}+1}{4}$$

$$\sin 36° = \sqrt{1-\cos^2 36°}$$

$$\sin 36° = \sqrt{1-\left(\frac{\sqrt{5}+1}{4}\right)^2}$$

$$\sin 36° = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

69. (C) cos 40 + cos 80 + cos 160

$$\Rightarrow \cos 40 + 2\cos\frac{80+160}{2}.\cos\frac{80-160}{2}$$

$$\left[\because \cos C+\cos D = 2\cos\frac{C+D}{2}.\cos\frac{C-D}{2}\right]$$

$$\Rightarrow \cos 40 + 2\cos 120 \,.\, \cos 40$$

$$\Rightarrow \cos 40 + 2\left(\frac{-1}{2}\right)\cos 40$$

$$\Rightarrow \cos 40 - \cos 40 = 0$$

70. (A) $\sin^{-1}\left(\sin\left(\frac{13\pi}{5}\right)\right)$

$$\Rightarrow \sin^{-1}\left[\sin\left(2\pi+\frac{3\pi}{5}\right)\right]$$

$$\Rightarrow \sin^{-1}\left(\sin\frac{3\pi}{5}\right)$$

$$\Rightarrow \sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{5}\right)\right]$$

$$\Rightarrow \sin^{-1}\left[\sin\left(\frac{2\pi}{5}\right)\right]$$

$$\Rightarrow \frac{2\pi}{5}$$

71. (C) $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$

$$y = \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$$

$$y = \tan\frac{x}{2}$$

$$\frac{dy}{dx} = \sec^2\frac{x}{2} \times \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{2\cos^2\frac{x}{2}}$$

$$\frac{dy}{dx} = \frac{1}{1+\cos x}$$

72. (B) $\frac{dy}{dx} - 3y = 1$

on comparing with general equation

$$\frac{dy}{dx} + Py = Q$$

where $P = -3$, $Q = 1$

$$I.F. = e^{\int P.dx}$$

$$= e^{\int -3dx}$$

$$I.F. = e^{-3x}$$

Solution of the differential equation

$$y \times I.F. = \int Q \times \text{I.F.}\, dx + c$$

$$y \times e^{-3x} = \int 1.e^{-3x}\, dx + c$$

$$y \times e^{-3x} = \frac{e^{-3x}}{-3} + c \qquad \text{...(i)}$$

given that $y(0) = 0$

$$0 \times e^0 = \frac{e^0}{-3} + c$$

$$c = \frac{1}{3}$$

from equation (i)

$$y \times e^{-3x} = -\frac{1}{3}e^{-3x} + \frac{1}{3}$$

$$y = \frac{-1}{3} + \frac{e^{3x}}{3}$$

$$y = \frac{e^{3x}-1}{3}$$

73. (B) $y = 2e^{3x}$

$$\frac{dy}{dx} = 2e^{3x} \times 3$$

$$m_1 = \left(\frac{dy}{dx}\right)_{\text{at}(0,2)} = 6$$

and slope of normal $m_2 = \frac{-1}{m_1}$

slope of normal $= \mathbf{\frac{-1}{6}}$

equation of normal at (0, 2)

$y - 2 = \frac{-1}{6}(x - 0)$

$x + 6y = 12$

74. (A) Equation of Normal

$x + 6y = 12$

this equation cuts the x-axis at $y = 0$

$x + 0 = 12$

$x = 12$

Normal to the curve $y = 2e^{3x}$ at (0, 2) cuts the x-axis at (12, 0).

75. (A) $xdy + ydx = 0$

$d(xy) = 0$

on integrating

$\int d(xy) = \int 0$

$xy = c$

76. (B) $f'(x) = x^3$

$z = fof(x)$

$z = f[f(x)]$

$z = f(x^3)$

$z = (x^3)^3$

$z = x^9$

$\frac{dz}{dx} = 9x^8$

$\frac{d^2z}{dx^2} = 72x^7$

77. (D) Vertex (0, 0) and focus (– 3, 0)

$a = 3$

equation of parabola

$y^2 = -4 \times 3x$

$y^2 + 12x = 0$

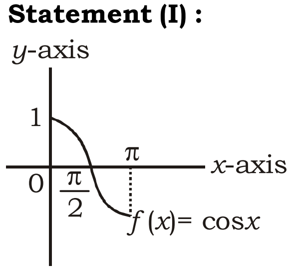
78. (B) The degree of the differential equation is not defined.

So statement I is incorrect.

and order = 2

Statement II is correct.

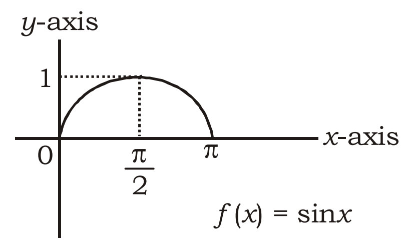
79.(D) **Statement (I) :**



$f(x) = \cos x$ decreases on the interval $\left(\frac{\pi}{2}, \pi\right)$.

So statement (I) is incorrect.

Statement (II) :



$f(x) = \sin x$ increases on the interval $\left(0, \frac{\pi}{2}\right)$.

So, statement (II) is incorrect.

80. (C)

81. (C) $\begin{vmatrix} 1+a & b+c-a & b+c \\ 1+b & c+a-b & c+a \\ 1+c & a+b-c & a+b \end{vmatrix}$

$C_2 \to C_2 - C_3$

$\Rightarrow \begin{vmatrix} 1+a & -a & b+c \\ 1+b & -b & c+a \\ 1+c & -c & a+b \end{vmatrix}$

$C_1 \to C_1 + C_2$

$\Rightarrow \begin{vmatrix} 1 & -a & b+c \\ 1 & -b & c+a \\ 1 & -c & a+b \end{vmatrix}$

$C_3 \to C_3 - C_2$

$\Rightarrow \begin{vmatrix} 1 & -a & a+b+c \\ 1 & -b & a+b+c \\ 1 & -c & a+b+c \end{vmatrix}$

$\Rightarrow -(a + b + c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$

$\Rightarrow -(a + b + c) \times 0 = 0$

[$\because$ two columns are identical.]

82. (A) **Statement (I) :**

$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{3}{5}$

$\Rightarrow \cos^{-1}\frac{4}{5} + \sin^{-1}\frac{4}{5}$

[$\because \cos^{-1} x = \sin^{-1}\sqrt{1 - x^2}$]

$\Rightarrow \frac{\pi}{2}$

Statement (I) is correct.

Statement II :

L.H.L. = $\cot^{-1}(\sqrt{3}) + \cot^{-1}(1)$

$$= \cot^{-1}\left(\frac{\sqrt{3}\times 1 - 1}{\sqrt{3}+1}\right)$$

$$\left[\because \cot^{-1}a + \cot^{-1}b = \cot^{-1}\left(\frac{ab-1}{a+b}\right)\right]$$

$$= \cot^{-1}\left(\frac{\sqrt{3}-1}{1+\sqrt{3}}\right)$$

$$= \cot^{-1}(2-\sqrt{3}) \neq \text{R.H.S.}$$

Statement (II) is incorrect.

83. (A) function $f : Z \to N$

where $Z = (0, \pm 1, \pm 2, \pm 3, \ldots)$

$N = (0, 1, 2, 3, 4 \ldots)$

$f(x) = |x|$

$x = 0, f(x) = 0$

$x = 1, f(x) = 1$

$x = -1 \;\; f(x) = 1$

$$\left.\begin{matrix}1\\-1\end{matrix}\right\rangle 1$$

Function is not one-one but function is onto.

84. (B) $z\bar{z} + (3i-1)z - (3i+1)\bar{z} + 4 = 0$

Let $z = x + iy$ and $\bar{z} = x - iy$

$\Rightarrow (x+iy)(x-iy) + (3i-1)(x+iy) - (3i+1)(x-iy) + 4 = 0$

$\Rightarrow x^2 + y^2 + 3ix - 3y - x - iy - 3ix - 3y - x + iy + 4 = 0$

$\Rightarrow x^2 + y^2 - 2x - 6y + 4 = 0$

$g = -1, f = -3, c = 4$

centre $(-g, -f) = (1, 3)$

radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{1+9-4}$

$= \sqrt{6}$

then centre (1, 3) and radius $\sqrt{6}$.

85. (C) Let $z = \dfrac{1}{1-\cos\theta - i\sin\theta}$

$$z = \frac{1}{(1-\cos\theta - i\sin\theta)} \times \frac{(1-\cos\theta)+i\sin\theta}{(1-\cos\theta)+i\sin\theta}$$

$$z = \frac{(1-\cos\theta)+i\sin\theta}{1+\cos^2\theta - 2\cos\theta - i^2\sin^2\theta}$$

$$z = \frac{1-\cos\theta + i\sin\theta}{2(1-\cos\theta)}$$

$$z = \frac{1}{2} + i\left(\frac{\sin\theta}{2(1-\cos\theta)}\right)$$

imaginary part of $z = \dfrac{\sin\theta}{2(1-\cos\theta)}$

86. (D) Equation

$bx^2 + cx + a = 0$

Let root = α, β

sum of roots $\alpha + \beta = \dfrac{-c}{b}$

product of roots $\alpha\beta = \dfrac{a}{b}$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$$= \frac{c^2}{b^2} - \frac{4a}{b}$$

$$(\alpha-\beta)^2 = \frac{c^2 - 4ab}{b^2}$$

$$(\alpha-\beta) = \sqrt{\frac{c^2-4ab}{b^2}}$$

according to question

$\alpha + \beta = \alpha^2 - \beta^2$

$(\alpha+\beta) = (\alpha-\beta)(\alpha+\beta)$

$\alpha - \beta = 1$

$$\sqrt{\frac{c^2-4ab}{b^2}} = 1$$

$c^2 - 4ab = b^2$

$b^2 + 4ab = c^2$

87. (A) equation

$bx^2 - ax + c = 0$

sum of roots $\alpha + \beta = \dfrac{a}{b}$

Product of roots $\alpha.\beta = \dfrac{c}{b}$

then $(b\alpha - a)(b\beta - a) = b^2\alpha\beta - ab\beta - ab\alpha + a^2$

$= b^2\alpha\beta - ab(\alpha+\beta) + a^2$

$= b^2 \times \dfrac{c}{b} - ab \times \dfrac{a}{b} + a^2$

$= bc - a^2 + a^2$

$= bc$

88. (D) The probabily that the problem is not be solved by any three students.

$$P(E) = \left(1-\frac{1}{3}\right)\left(1-\frac{2}{5}\right)\left(1-\frac{1}{6}\right)$$

$= \frac{2}{3} \times \frac{3}{5} \times \frac{5}{6}$

$= \frac{1}{3}$

89. (D) A coin tossed 6 times

total number of ways = 2^6

If head comes odd times

The number of ways = $\frac{2^6}{2}$

$= 2^5$

then probability $P(E) = \frac{2^5}{2^6} = \frac{1}{2}$

90. (A)

2	73	
2	36	1
2	18	0
2	9	0
2	4	1
2	2	0
2	1	0
	0	1

$(73)_{10} = (1001001)_2$

0.625
×2
[1].250
×2
[0].500
×2
[1].000

$(0.625)_{10} = (0.101)_2$

then $(73.625)_0 = (1001001.101)_2$

91. (C) Given that $f(x) = \sqrt{36 - x^2}$

$$f'(x) = \frac{-x}{\sqrt{36 - x^2}}$$

$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$ $\left[\frac{0}{0}\right]$ Form

$\Rightarrow \lim_{x \to 2} \frac{f'(x) - 0}{1 - 0}$

$\Rightarrow \lim_{x \to 2} \frac{-x}{\sqrt{36 - x^2}}$

$\Rightarrow \frac{-2}{\sqrt{32}}$

$\Rightarrow \frac{-2}{4\sqrt{2}}$

$\Rightarrow \frac{-1}{2\sqrt{2}}$

92. (B) Given that $f(x) = b^3$

$\lim_{x \to 2} f\left(\frac{x}{b^2}\right) = \lim_{x \to 2} b^3$

$= b^3$

93. (A) Given $f(x) = \begin{cases} \pi^2 \cos x \text{ for } x \in [-\pi, 0) \\ (x + \pi)^2 \text{ for } x \in \left[0, \frac{\pi}{2}\right] \end{cases}$

Statement (I) :

L.H.L. $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h)$

$= \lim_{h \to 0} \pi^2 (0 - h)$

$= \pi^2$

R.H.L. $= \lim_{h \to 0^+} f(x) = \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} [0 + h + \pi]^2$

$= \pi^2$

L.H.L. = R.H.L.

function $f(x)$ is continuous at $x = 0$

Statement (I) is correct.

Statement II :

L.H.D. $= \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$

$= \lim_{h \to 0} \frac{\pi^2 \cos(0 - h) - \pi^2}{-h}$

$= \lim_{h \to 0} \frac{\pi^2 \cos h - \pi^2}{-h}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$\Rightarrow \lim_{h \to 0} \frac{-\pi^2 \sin h - 0}{-1}$

$\Rightarrow 0$

R.H.D. $= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h \to 0} \frac{(0 + h + \pi)^2 - \pi^2}{h}$

$= \lim_{h\to0} \dfrac{h^2+\pi^2+2h\pi-\pi^2}{h}$

$= \lim_{h\to0} h + 2\pi$

$= 2\pi$

L.H.D. $\neq$ R.H.D.

function is not differentiable at $x = 0$

Statement (II) is incorrect.

94. (B) $f(x) = \dfrac{a^{[x]-x}-1}{[x]-x}$

R.H.L. $= \lim_{x\to0^+} f(x) = \lim_{h\to0} f(0+h)$

$= \lim_{h\to0} \dfrac{a^{[0+h]-0-h}-1}{[0+h]-0-h}$

$= \lim_{h\to0} \dfrac{a^{-h}-1}{-h}$ $\left[\dfrac{0}{0}\right]$ form

$= \lim_{h\to0} \dfrac{-1\times a^{-h}\log a-0}{-1}$

$= \dfrac{-1\times a^0\times\log a}{-1}$

$= \log a$

95. (C) L.H.L. $= \lim_{x\to0^-} f(x) = \lim_{h\to0} f(0-h)$

$= \lim_{h\to0} \dfrac{a^{[0-h]-0+h}-1}{[0-h]-0+h}$

$= \lim_{h\to0} \dfrac{a^{-1+h}-1}{-1+h}$

$= \dfrac{a^{-1}-1}{-1} = 1-\dfrac{1}{a}$

96. (A) $\lim_{x\to0} x\cot x$

$\Rightarrow \lim_{x\to0} \dfrac{x}{\tan x}$ $\left[\dfrac{0}{0}\right]$ Form

$\Rightarrow \lim_{x\to0} \dfrac{1}{\sec^2 x}$

$= \dfrac{1}{\sec^2 0}$

$= 1$

97. (C) $I = \int\sqrt{1+\sin\dfrac{x}{8}}\,dx$

$I = \int\sqrt{\sin^2\dfrac{x}{16}+\cos^2\dfrac{x}{16}+2\sin\dfrac{x}{16}.\cos\dfrac{x}{16}}\,dx$

$I = \int\sqrt{\left(\sin\dfrac{x}{16}+\cos\dfrac{x}{16}\right)^2}\,dx$

$I = \int\left(\sin\dfrac{x}{16}+\cos\dfrac{x}{16}\right)dx$

$= \dfrac{-\cos\dfrac{x}{16}}{\dfrac{1}{16}} + \dfrac{\sin\dfrac{x}{16}}{\dfrac{1}{16}} + c$

$I = 16\left(\sin\dfrac{x}{16}-\cos\dfrac{x}{16}\right) + c$

98. (C) $I = \int e^x\left(\dfrac{1-\sin x}{1-\cos x}\right)dx$

$I = \int e^x\left(\dfrac{1}{1-\cos x}-\dfrac{\sin x}{1-\cos x}\right)dx$

$I = \int e^x\left(\dfrac{1}{2\sin^2\dfrac{x}{2}}-\dfrac{2\sin\dfrac{x}{2}.\cos\dfrac{x}{2}}{2\sin^2\dfrac{x}{2}}\right)dx$

$I = \int e^x\left(\dfrac{1}{2}\text{cosec}^2\dfrac{x}{2}-\cot\dfrac{x}{2}\right)dx$

$I = -\int e^x\left(\cot\dfrac{x}{2}-\dfrac{1}{2}\text{cosec}^2\dfrac{x}{2}\right)dx$

$I = -e^x\cot\dfrac{x}{2} + c$

$\left[\int e^x.(f(x)+f'(x))dx = e^x f(x)+c\right]$

$I = -e^x \times \dfrac{\cos\dfrac{x}{2}}{\sin\dfrac{x}{2}} + c$

99. (A) $I = \int 17^x\,dx$

$I = \dfrac{17^x}{\log_e 17} + c$ $\left[\because \int a^x dx = \dfrac{a^x}{\log_e c}+c\right]$

100. (B) $I_n = \int (\sin x)^n$ ...(i)

$I_{n-2} = \int (\sin x)^{n-2}\, dx$ ...(ii)

$I_n = \int (\sin x)^{n-1} .\sin x\, dx$

$= (\sin x)^{n-1} \int \sin x dx - \int \left\{ \frac{d}{dx} (\sin x)^{n-1} \int \sin x\, dx \right\} dx$

$= -(\sin x)^{n-1}.\cos x -$

$\int (n-1)(\sin x)^{n-2} .\cos x.(-\cos x)\, dx$

$I_n = -\cos x (\sin x)^{n-1} + (n-1) \int (\sin x)^{n-2} \cos^2 x\, dx$

$I_n = -\cos x(\sin x)^{n-1} + (n-1) \int (\sin x)^{n-2}$

$(1 - \sin^2 x)\, dx$

$I_n = -\cos x (\sin x)^{n-1} + (n-1) \int (\sin x)^{n-2}\, dx$

$-(n-1) \int (\sin x)^n\, dx$

$I_n = -\cos x(\sin x)^{n-1} + (n-1) I_{n-2} - (n-1)I_n$

$nI_n = -\cos x (\sin x)^{n-1} + (n-1)I_{n-2}$

$nI_n - (n-1)I_{n-2} = -\cos x (\sin x)^{n-1}$

101. (C) $I = \int_0^{\frac{\pi}{4}} \cos^3 2x\, dx$

$I = \int_0^{\frac{\pi}{4}} \cos 2x \cos^2 2x\, dx$

$I = \int_0^{\frac{\pi}{4}} \cos 2x (1-\sin^2 2x)\, dx$

$I = \int_0^{\frac{\pi}{4}} \cos 2x\, dx - \int_0^{\frac{\pi}{4}} \cos 2x .\sin^2 2x\, dx$

Let $\sin 2x = t$ when $x \to 0,\ t \to 0$

$2\cos 2x\, dx = dt$ $x \to \frac{\pi}{4},\ t \to 1$

$\cos 2x\, dx = \frac{1}{2} dt$

$I = \left[\frac{\sin 2x}{2} \right]_0^{\frac{\pi}{4}} - \int_0^1 t^2 \frac{1}{2} dt$

$I = \frac{1}{2} \left[\sin\frac{\pi}{2} - \sin 0 \right] - \frac{1}{2} \left[\frac{t^3}{3} \right]_0^1$

$I = \frac{1}{2} [1-0] - \frac{1}{2} \left[\frac{1}{3} - 0 \right]$

$I = \frac{1}{2} - \frac{1}{6}$

$I = \frac{1}{3}$

102. (B) $I = \int_0^{\pi} e^{\sin x} \cos^5 x\, dx$ ...(i)

$I = \int_0^{\pi} e^{\sin(\pi - x)} \cos^5 (\pi - x)\, dx$ (Pro. IV)

$I = \int_0^{\pi} e^{\sin(\pi - x)} \cos^5 x\, dx$ ...(ii)

from equation (i) and equation (ii)

$2I = \int 0\, dx$

$I = 0$

103. (A) $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x\, dx$

$I_n + I_{n-2} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{n-2} x\, dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{n-2} x (\cot^2 x + 1)\, dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{cosec}^2 x.(\cot x)^{n-2}\, dx$

Let $\cot x = t$ when $x \to \frac{\pi}{4},\ t \to 1$

$-\operatorname{cosec}^2 x\, dx = dt$ $x \to \frac{\pi}{2},\ t \to 0$

$\operatorname{cosec}^2 x\, dx = -dt$

$I_n + I_{n-2} = \int_1^0 -t^{n-2}\, dt$

$= \int_0^1 t^{n-2}\, dt$

$= \left[\frac{t^{n-2+1}}{n-2+1} \right]_6^1$

$= \left[\frac{1}{n-1} - 0 \right]$

$I_n + I_{n-2} = \frac{1}{n-1}$

104. (C) $I = \int_0^1 e^{-x} (x^2 - 2x)\, dx$

Let $-x = t$ where $x \to 0,\ t = 0$

$-dx = dt$ $x = 1,\ t = -1$

$dx = -dt$

$I = -\int_0^{-1} e^t (t^2 + 2t)\, dt$

$I = -\left[e^t\, t^2 \right]_0^{-1}$

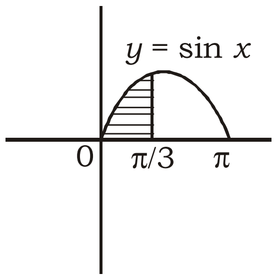
$$\left[\because \int e^x \left(f(x) + f'(x)\right)dx = e^x . f(x) + c\right]$$

$$I = -\left[e^{-1}(-1)^2 - e^0 \times 0\right]$$

$$I = -\left[e^{-1}\right]$$

$$I = -\frac{1}{e}$$

105. (A)



curve $y_1 \Rightarrow y = \sin x$

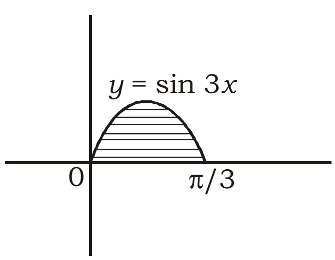
$$A_1 = \int_0^{\frac{\pi}{3}} y_1 \, dx$$

$$A_1 = \int_0^{\frac{\pi}{3}} \sin x \, dx$$

$$= -\left[\cos x\right]_0^{\frac{\pi}{3}}$$

$$= -\left[\cos\frac{\pi}{3} - \cos 0\right]$$

$$A_1 = -2[-1/2 - 1] = \frac{1}{2}$$



curve $y^2 \Rightarrow y = \sin 3x$

$$A_2 = \int_0^{\frac{\pi}{3}} y_2 \, dx$$

$$A_2 = \int_0^{\frac{\pi}{3}} \sin 3x \, dx$$

$$A_2 = -\left[\frac{\cos 3x}{3}\right]_0^{\frac{\pi}{3}}$$

$$= -\frac{1}{3}\left[\cos \pi - \cos 0\right]$$

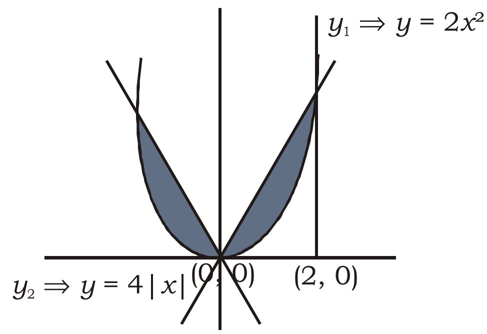
$$= -\frac{1}{3}[-1-1]$$

$$A_2 = \frac{2}{3}$$

$$A_1 : A_2 = \frac{1}{2} : \frac{2}{3}$$

$$= 3 : 4$$

106. (B)



curves

$y_1 \Rightarrow y = 2x^2$ ...(i)

and $y_2 \Rightarrow y = 4|x|$ ...(ii)

$$y_2 \Rightarrow y = \begin{cases} 4x, & x > 0 \\ -4x, & x < 0 \end{cases}$$

from equation (i) and equation (ii)

$x = 0,\quad x = 2,\quad x = -2$

$y = 0,\quad y = 8\quad y = 8$

$$\text{Area} = 2\int_0^2 (y_2 - y_1)\,dx$$

$$= 2\int_0^2 (4x - 2x^2)\,dx$$

$$= 2\left[\frac{4x^2}{2} - \frac{2x^3}{3}\right]_0^2$$

$$= 2\left[\frac{4\times 4}{2} - \frac{2\times 8}{3} = 0\right]$$

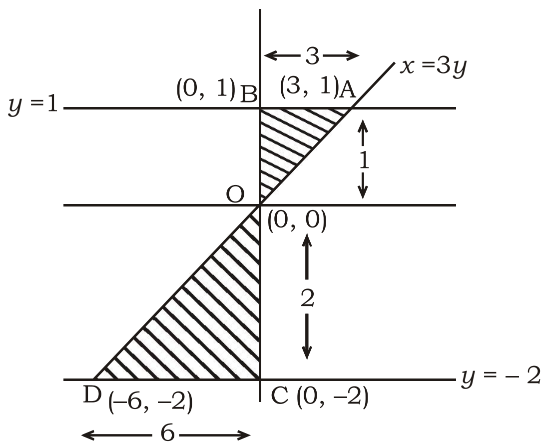
$$\text{Area} = 2\left[8 - \frac{16}{3}\right]$$

$$= 2 \times \frac{8}{3}$$

$$= \frac{16}{3} \text{ square unit.}$$

107. (A) line $x_1 \Rightarrow x = 3y$

$y = 1$ and $y = -2$



Area = Area of ΔAOB + Area of ΔCOD

$= \frac{1}{2} \times 1 \times 3 + \frac{1}{2} \times 6 \times 2$

$= \frac{3}{2} + 6$

$= \frac{15}{2}$ square unit

108. (C) $3x + 4y = 8$ ...(i)

$4x - 3y = \frac{13}{2}$ ...(ii)

$x = 0$ ...(iii)

equation (i) and equation (ii) are perpendicular to each other.

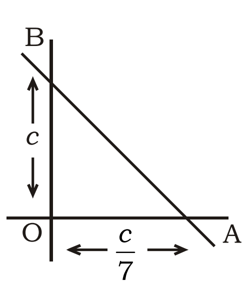
So the triangle is right-angled triangle.

109. (B) Given line $7x + y = 5$

equation of line which is parallel to given line

$7x + y = c$ ...(i)

$\frac{x}{\frac{c}{7}} + \frac{y}{c} = 1$ ...(ii)



Area $= \frac{1}{2} \times OA \times OB$

$14 = \frac{1}{2} \times \frac{c}{7} \times c$

$c^2 = (14)^2$

$c = \pm 14$

from equation (i)

$7x + y = \pm 14$

110. (A) Equations

$x^2 + y^2 = 9$ ...(i)

and $x + y = -3$ ...(ii)

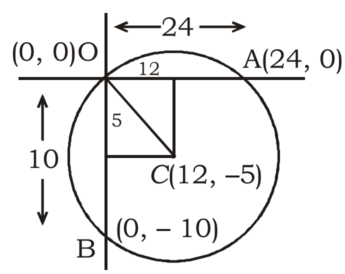
from equation (i) and equation (ii)

$x = 0,$ $x = -3$

$y = -3,$ $y = 0$

set is {(0, – 3), (– 3, 0)}.

111. (C)



Equation of circle whose centre (12, –5)

$(x - 12)^2 + (y + 5)^2 = r^2$ ...(i)

passing through the point (0, 0)

$144 + 25 = r^2$

$r = 13$

equation of circle

$(x - 12)^2 + (y + 5)^2 = 169$ ...(ii)

intercepts on x-axis and

y-axis is OA = 24 unit and OB = 10 unit

112. (D) Equation $7x - 8y = 7$

Equation of line which is perpendicular to the given line

$8x + 7y = c$ ...(i)

equation (i) passing through the point (–2, 0) then

$-16 + 7 \times 0 = c$

$c = -16$

from equation (i)

$8x + 7y = -16$

$8x + 7y + 16 = 0$

113. (D) Given numbers

9, 15, 8, 17, 21, 30

on arranging in ascending order

8, 9, 15, 17, 21, 30

$n = 6$

Median (A)= $\dfrac{\left(\dfrac{n}{2}\right)^{th} \text{term} + \left(\dfrac{n}{2}+1\right)^{th} \text{term}}{2}$

$= \dfrac{\text{3rd term+ 4th term}}{2}$

$(A) = \dfrac{15+17}{2} = 16$

Mean deviation = $\dfrac{\sum|x_i - A|}{n}$

$= \dfrac{|9-16|+|15-16|+|8-16|+|17-16|+|21-16|+|30-16|}{6}$

$= \dfrac{7+1+8+1+5+14}{6} = \dfrac{36}{6} = 6$

114. (C)

115. (B) **From option (B)** vector $\dfrac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}$

angle between $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\dfrac{(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{6}}$

= 90°

and angle between $(-2\hat{i} + 2\hat{j} + 6\hat{k})$ and $\dfrac{(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{6}}$ = 90°

unit vector $\dfrac{(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{6}}$ is orthogonal to the vector $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $(-2\hat{i} + 2\hat{j} + 6\hat{k})$.

116. (D) We know that

$|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$

and $|\vec{a} - \vec{b}| \ge |\vec{a}| - |\vec{b}|$

117. (C) The equation of sphere passing through $x^2 + y^2 + z^2 = 9$, $3x - y + 7z = 5$ is given by

$(x^2 + y^2 + z^2 - 9) + k(3x - y + 7z - 5) = 0$ ...(i)

equation (i) passing through the point (– 1, 2, 1) then

$1 + 4 + 1 - 9 + k(-3 - 2 + 7 - 5) = 0$

$k = -1$

from equation (i)

equation of sphere

$x^2 + y^2 + z^2 - 9 - 3x + y - 7z + 5 = 0$

$x^2 + y^2 + z^2 - 3x + y - 7z - 4 = 0$

118. (B) line $\dfrac{x}{2} = \dfrac{y}{\sqrt{3}} = \dfrac{z}{3}$

$a_1 = 2,\ b_1 = \sqrt{3}{}_1\ c_1 = 3$

and plane $4x - 3z + 4 = 0$

$a_2 = 4,\ b_2 = 0,\ c_2 = -3$

angle between line and plane

$\sin\theta = \left|\dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$

$= \left|\dfrac{2\times(4) + \sqrt{3}\times 0 + 3\times(-3)}{\sqrt{(2)^2 + (\sqrt{3})^2 + (3)^2}\sqrt{(4)^2 + (-3)^2}}\right|$

$= \left|\dfrac{-1}{4\times 5}\right|$

$\sin\theta = \dfrac{1}{20}$

$\theta = \sin^{-1}\left(\dfrac{1}{20}\right)$

119. (A) Equation of the line passing through the points (– 1, 2, 0) and (– 3, – 4, 5)

$\dfrac{x+3}{-1+3} = \dfrac{y+4}{2+4} = \dfrac{z-5}{0-5}$

$\dfrac{x+3}{2} = \dfrac{y+4}{6} = \dfrac{z-5}{-5}$

120. (C)

class	f	c	
0 – 10	5	5	
10 – 20	6	11	
20 – 30	17	28	
30 - 40	**20**	**48**	Median class
40 – 50	9	57	
50 – 60	13	60	

$n = 60$

$\dfrac{n}{2} = \dfrac{60}{2} = 30$

$l_1 = 30,\ l_2 = 40,\ f = 20,\ C = 28$

median = $l_1 + \dfrac{\dfrac{n}{2} - C}{f} \times (l_2 - l_1)$

$= 30 + \dfrac{30-28}{20} \times (40-30)$

$= 30 + \dfrac{2}{20} \times 10$

median = 31

NDA (MATHS) MOCK TEST - 78 (Answer Key)

1.	(C)	21.	(D)	41.	(C)	61.	(D)	81.	(C)	101.	(C)
2.	(B)	22.	(B)	42.	(B)	62.	(C)	82.	(A)	102.	(B)
3.	(A)	23.	(B)	43.	(D)	63.	(C)	83.	(A)	103.	(A)
4.	(A)	24.	(C)	44.	(B)	64.	(B)	84.	(B)	104.	(B)
5.	(C)	25.	(A)	45.	(B)	65.	(A)	85.	(C)	105.	(C)
6.	(B)	26.	(C)	46.	(C)	66.	(C)	86.	(D)	106.	(B)
7.	(B)	27.	(D)	47.	(B)	67.	(B)	87.	(A)	107.	(A)
8.	(A)	28.	(A)	48.	(A)	68.	(B)	88.	(D)	108.	(C)
9.	(D)	29.	(B)	49.	(C)	69.	(C)	89.	(D)	109.	(B)
10.	(B)	30.	(B)	50.	(B)	70.	(A)	90.	(A)	110.	(D)
11.	(C)	31.	(C)	51.	(D)	71.	(C)	91.	(C)	111.	(C)
12.	(C)	32.	(A)	52.	(C)	72.	(B)	92.	(B)	112.	(D)
13.	(C)	33.	(B)	53.	(C)	73.	(B)	93.	(A)	113.	(D)
14.	(B)	34.	(C)	54.	(B)	74.	(A)	94.	(B)	114.	(C)
15.	(A)	35.	(A)	55.	(C)	75.	(A)	95.	(C)	115.	(B)
16.	(C)	36.	(A)	56.	(A)	76.	(B)	96.	(A)	116.	(D)
17.	(A)	37.	(C)	57.	(C)	77.	(D)	97.	(C)	117.	(C)
18.	(C)	38.	(B)	58.	(C)	78.	(B)	98.	(C)	118.	(B)
19.	(A)	39.	(B)	59.	(A)	79.	(D)	99.	(A)	119.	(A)
20.	(C)	40.	(C)	60.	(A)	80.	(C)	100.	(B)	120.	(C)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*