NDA MATHS MOCK TEST - 80 (SOLUTION)

1. (C) $nRm \Leftrightarrow n$ is divisible by m

Reflexive :

$nRn \Leftrightarrow n$ is divisible by n

So R is Reflexive.

Symmetric :

$nRm \Leftrightarrow n$ is divisible by m

but $mRn \Leftrightarrow m$ will not divisible by n

So R is not symmetric.

Transitive :

$nRm \Leftrightarrow n$ is divisible by m

$mRl \Leftrightarrow m$ will divisible by l

then $nRl \Leftrightarrow m$ is also divisible by l.

So R is transitive.

2. (C) $\begin{vmatrix} b & c & a \\ a & b & c \\ c & a & b \end{vmatrix}$

$\Rightarrow b(b^2 - ca) - c(ab - c^2) + a(a^2 - bc)$

$\Rightarrow b^3 - abc - abc + c^3 + a^3 - abc$

$\Rightarrow a^3 + b^3 + c^3 - 3abc$

$\Rightarrow$ positive

3. (B) given that

$f(x) = 3x + 7$

x	$f(x)$
1	10
2	13
3	16
⋮	⋮

so on

So function injective but not surjective.

4. (B) 6 letters can be posted in 7 letter boxes in $= 7^6$

5. (C) Given that

$\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} A = \begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}$

from option (C)

$\begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ 1 & -1 \end{bmatrix}$ satisfy the equation.

L.H.S $= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} A$

$= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 0+1 & 0-1 \\ 3-3 & 1+3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}$ = R.H.S

then $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ 1 & -1 \end{bmatrix}$

6. (B) $\log_e(x + \sqrt{x^2-1}) + \log_e(x - \sqrt{x^2-1})$

$\Rightarrow \log_e[(x + \sqrt{x^2-1})(x - \sqrt{x^2-1})]$

$[\because \log m + \log_e n = \log_e mn]$

$\Rightarrow \log_e [x^2 - (x^2 - 1)]$

$\Rightarrow \log_e 1$

$\Rightarrow 0$

7. (B) given $f(2x) = \dfrac{x+1}{x-1}$

Let $2x = y \Rightarrow x = \dfrac{y}{2}$

$f(y) = \dfrac{\frac{y}{2}+1}{\frac{y}{2}-1}$

$f(y) = \dfrac{y+2}{y-2}$

$f(x) = \dfrac{x+2}{x-2}$

from option (B)

$\dfrac{3f(2x)-1}{3-f(2x)} = \dfrac{3\left(\frac{x+1}{x-1}\right)-1}{3-\frac{x+1}{x-1}}$

$= \dfrac{3x+3-x+1}{3x-3-x-1}$

$= \dfrac{2x+4}{2x-4}$

$= \dfrac{2(x+2)}{2(x-2)}$

$\dfrac{3f(2x)-1}{3-f(2x)} = f(x)$

8. (C) $f(x) = \dfrac{x+2}{x-2}$

then $\dfrac{f(x)+1}{f(x)-1} - x = \dfrac{\dfrac{x+2}{x-2}+1}{\dfrac{x+2}{x-2}-1} - x$

$= \dfrac{2x}{4} - x$

$= -\dfrac{2x}{4} = -\dfrac{x}{2}$

9. (A) $f(x) = \dfrac{x+2}{x-2}$

$f(f(x)) = \dfrac{\dfrac{x+2}{x-2}+2}{\dfrac{x+2}{x-2}-2}$

$= \dfrac{3x-2}{-x+6}$

$= \dfrac{3x-2}{6-x}$

10. (B) We know that

$\cos A \cos(60^\circ - A)\cos(60^\circ + A) = \dfrac{1}{4}\cos 3A$

then $\sqrt{k} = \dfrac{1}{4}$

$k = \dfrac{1}{16}$

11. (C) $x = a(\cos\theta + \theta\sin\theta)$

$\dfrac{dx}{d\theta} = a[-\sin\theta + \theta.\cos\theta + \sin\theta.1]$

$\dfrac{dx}{d\theta} = a\theta.\cos\theta$

and $y = a(-\sin\theta + \theta\cos\theta)$

$\dfrac{dy}{d\theta} = a(-\cos\theta + \theta(-\sin\theta) + \cos\theta)$

$\dfrac{dy}{d\theta} = -a\theta\sin\theta$

$\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \times \dfrac{d\theta}{dx}$

$= \dfrac{-a\theta\sin\theta}{a\theta\cos\theta}$

$= -\tan\theta$

12. (C)

```
      1001.0
     ×110.1
      10010
     00000
    10010
   10010
   111010.10
```

Product of two numbers = $(111010.1)_2$

13. (D) $\omega^{299} + \omega^{300} + \omega^{301}$

$\Rightarrow \omega^{299}(1 + \omega + \omega^2)$

$\Rightarrow \omega^{299} \times 0 \qquad [\because 1 + \omega + \omega^2 = 0]$

$\Rightarrow 0$

14. (C) $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{3, 4, 7\}$, $B = \{2, 4, 6\}$, $C = \{2, 4, 7\}$

then $(A \cup C) = \{(3, 4, 7) \cup (2, 4, 7)\}$
$= \{2, 3, 4, 7\}$

$(A \cup B) = \{(3, 4, 7) \cup (2, 4, 6)\}$
$= \{2, 3, 4, 6, 7\}$

$(A \cup C) \cap (A \cup B) = \{2, 3, 4, 7\}$

$B - \{(A \cup C) \cap (A \cup B)\} = \{2, 4, 6\} - \{2, 3, 4, 7\}$
$= \{6\}$

15. (A) $f(x) = \sqrt{2x - x^2}$

Let $f(x) = y$

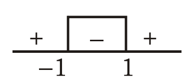
$y = \sqrt{2x - x^2}$

$y^2 = 2x - x^2$

$x^2 - 2x + y^2 = 0$

$x = \dfrac{+2 \pm \sqrt{4 - 4y^2}}{2 \times 1}$

$x = \dfrac{2 \pm \sqrt{4 - 4y^2}}{2}$



$4 - 4y^2 \geq 0$

$y^2 - 1 \leq 0$

$(y - 1)(y + 1) \leq 0$

$-1 \leq y \leq 1$

Range of $f(x) = [-1, 1]$

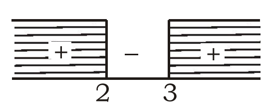
16. (B) $f(x) = \sqrt{\log(x^2 - 5x + 7)}$

$\log(x^2 - 5x + 7) \geq 0$

$x^2 - 5x + 7 \geq 1$

$x^2 - 5x + 6 \geq 0$

$(x - 2)(x - 3) \geq 0$



$x \in (-\infty, 2] \cup [3, \infty)$

17. (C) $\lim_{x\to 3} \frac{27-x^3}{x-3}\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 3} \frac{0-3x^2}{1-0}$

$\Rightarrow \lim_{x\to 3} -3x^2$

$\Rightarrow -3 \times (3)^2$

$\Rightarrow -27$

18. (A) Let $a - ib = \sqrt{1-4\sqrt{5}i}$

On squaring both side

$a^2 - b^2 - 2abi = 1 - 4\sqrt{5}\,i$

$a^2 - b^2 = 1 \quad 2ab = 4\sqrt{5}$ ...(i)

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$

$(a^2 + b^2)^2 = 1 + 80$

$a^2 + b^2 = 9$ ...(ii)

from equation (i) and equation (ii)

$2a^2 = 10$ and $2b^2 = 8$

$a^2 = 5 \quad b^2 = 4$

$a = \pm\sqrt{5} \quad b = \pm 2$

$\sqrt{1-4\sqrt{5}i} = \pm(\sqrt{5} - 2i)$

square root of $(1 - 4\sqrt{5}\,i) = \pm(\sqrt{5} - 2i)$

19. (C) **Statement (I) :**

$(\omega^{13} + 1)^{11} + \omega = (\omega + 1)^{11} + \omega$

$= (-\omega^2)^{11} + \omega$

$= -\omega^{22} + \omega$

$= -\omega + \omega$

$(\omega^{13} + 1)^{11} + \omega = 0$

Statement I is correct.

Statement (II) :

$(\omega^{174} + 1)^{15} = ((\omega^3)^{58} + 1)^{15}$

$(\omega^{174} + 1)^{15} = (1 + 1)^{15}$

$= 2^{15}$

Statement II is correct.

20. (B)

21. (B) $A = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$

$AB = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$

$AB = C$

22. (D) Equation

$x^2 + 3x + 2 = 0$

$\alpha + \beta = -3$

$\alpha \times \beta = 2$

then $\frac{\alpha^6 - \beta^6}{\alpha^{-6} - \beta^{-6}} = \frac{\alpha^6 - \beta^6}{\frac{1}{\alpha^6} - \frac{1}{\beta^6}}$

$= \frac{(\alpha^6 - \beta^6)\alpha^6\beta^6}{\beta^6 - \alpha^6}$

$= -(\alpha\beta)^6$

$= -2^6$

$= -64$

23. (C) $\frac{\alpha}{x+\alpha} + \frac{\beta}{x+\beta} = -1$

$\frac{x\alpha + \beta\alpha + \beta x + \alpha\beta}{x^2 + x(\alpha+\beta) + \alpha\beta} = -1$

$x(\alpha + \beta) + 2\alpha\beta = -x^2 - x(\alpha + \beta) - \alpha\beta$

$x^2 + 2x(\alpha + \beta) + 3\alpha\beta = 0$

Let roots $a, -a$

$a - a = -2(\alpha + \beta)$

$\alpha + \beta = 0$

24. (C) $x^2 - 17x + 30 = 0$

$(x - 15)(x - 2) = 0$

$x = 2, 15$

So roots are distinct and real.

25. (A)

26. (B) $\frac{a^{n+3} + b^{n+3}}{a^{n+2} + b^{n+2}}$

On putting $\boldsymbol{n = -2}$

$\Rightarrow \frac{a^{-2+3} + b^{-2+3}}{a^{-2+2} + b^{-2+2}}$

$\Rightarrow \frac{a+b}{2}$ = G.M of a and b.

27. (C) $\log_7 5, \log_{35} 5, \log_{175} 5$

$\frac{1}{\log_5 7}, \frac{1}{\log_5 35}, \frac{1}{\log_5 175}$

Let series $\log_5 7$, $\log_5 35$, and $\log_5 175$ are in A.P.

then $2\log_5 35 = \log_5 7 + \log_5 175$

$\log_5 (35)^2 = \log_5 (7\times175)$

$1225 = 1225$

Series $\log_5 7$, $\log_5 35$ and $\log_5 175$ are in A.P.

Hence $\frac{1}{\log_5 7}, \frac{1}{\log_5 35}, \frac{1}{\log_5 175}$ are in H.P.

$\log_7 5, \log_{35} 5, \log_{175} 5$ are in H.P.

28. (A) Let $y = 5^{83}$

taking log

$\log_{10} y = 83\log_{10} 5$

$= 83 \times 0.699$

$= 58.077$

The number of digits in $5^{83} = 59$

29. (C) $\frac{1}{1+\log_b a+\log_b c} + \frac{1}{1+\log_c a+\log_c b} + \frac{1}{1+\log_a b+\log_a c}$

$\Rightarrow \frac{1}{\log_b b+\log_b a+\log_b c} + \frac{1}{\log_c c+\log_c a+\log_c b} + \frac{1}{\log_a a+\log_a b+\log_a c}$

$\Rightarrow \frac{1}{\log_b abc} + \frac{1}{\log_c abc} + \frac{1}{\log_a abc}$

$\Rightarrow \log_{abc} b + \log_{abc} c + \log_{abc} a$

$\Rightarrow \log_{abc} abc$

$\Rightarrow 1$

30. (C) $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \end{vmatrix}$

$R_3 \to R_3 - R_2$

$\Rightarrow \begin{vmatrix} a^2 & b^2 & c^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \\ 4a & 4b & 4c \end{vmatrix}$

$\Rightarrow 4\begin{vmatrix} a^2 & b^2 & c^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \\ a & b & c \end{vmatrix}$

$R_2 \to R_2 - R_1 + 2R_3$

$\Rightarrow 4\begin{vmatrix} a^2 & b^2 & c^2 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$

$\Rightarrow -4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

31. (C) Equations

$x + 2y + z = 3$

$4x + y + 2z = 2$

$-4x + y - z = 3$

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 1 & 2 \\ -4 & 1 & -1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$

$AX = B$

$A^{-1} AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$ ...(i)

$|A| = 1(-1-2) - 2(-4+8) + 1(4+4)$

$= -3 - 8 + 8$

$|A| = -3$

Co-factors of A –

$C_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}$, $C_{12} = (-1)^{1+2}\begin{vmatrix} 4 & 2 \\ -4 & -1 \end{vmatrix}$, $C_{13} = (-1)^{1+3}\begin{vmatrix} 4 & 1 \\ -4 & 1 \end{vmatrix}$

$= -3$ $\quad = -4$ $\quad = 8$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$, $C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ -4 & -1 \end{vmatrix}$, $C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ -4 & 1 \end{vmatrix}$

$= 3$ $\quad = 3$ $\quad = -9$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$, $C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix}$, $C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix}$

$= 3$ $\quad = 2$ $\quad = -7$

$C = \begin{bmatrix} -3 & -4 & 8 \\ 3 & 3 & -9 \\ 3 & 2 & -7 \end{bmatrix}$

$\text{adj } A = C^T = \begin{bmatrix} -3 & 3 & 3 \\ -4 & 3 & 2 \\ 8 & -9 & -7 \end{bmatrix}$

$A^{-1} = \frac{\text{adj} A}{|A|} = \begin{bmatrix} 1 & -1 & -1 \\ \frac{4}{3} & -1 & \frac{-2}{3} \\ -\frac{8}{3} & 3 & \frac{7}{3} \end{bmatrix}$

from equation (i)

$X = A^{-1} B$

$= \begin{bmatrix} 1 & -1 & -1 \\ \frac{4}{3} & -1 & -\frac{2}{3} \\ -\frac{8}{3} & 3 & \frac{7}{3} \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$

$$X = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$$

$x = -2,\ y = 0,\ z = 5$

32. (A) $\dfrac{\cos(x+y)}{\cos(x-y)} = \dfrac{a-b}{a+b}$

by componendo and dividendo rule

$$\frac{\cos(x+y)+\cos(x-y)}{\cos(x+y)-\cos(x-y)} = \frac{a-b+a+b}{a-b-a-b}$$

$$\frac{2\cos\frac{x+y+x-y}{2}.\cos\frac{x+y-x+y}{2}}{2\sin\frac{x+y+x-y}{2}.\sin\frac{x-y-x-y}{2}} = \frac{2a}{-2b}$$

$$\frac{\cos x.\cos y}{\sin x(-\sin y)} = \frac{a}{-b}$$

$$\frac{\cot x}{\tan y} = \frac{a}{b}$$

33. (B) sin A + cos C = cos B

sin A = cos B – cos C

$$2\sin\frac{A}{2}.\cos\frac{A}{2} = 2\sin\frac{B+C}{2}.\sin\frac{C-B}{2}$$

$$\sin\frac{A}{2}.\cos\frac{A}{2} = \sin\left(90-\frac{A}{2}\right)\ \sin\frac{C-B}{2}$$

$$\sin\frac{A}{2}.\cos\frac{A}{2} = \cos\frac{A}{2}.\sin\frac{C-B}{2}$$

$$\sin\frac{A}{2} = \sin\frac{C-B}{2}$$

$$\frac{A}{2} = \frac{C-B}{2}$$

$A = C - B$

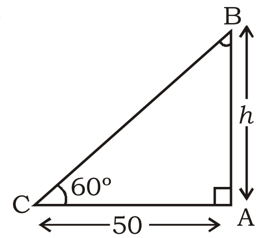
$A + B = C$

$180 - C = C$

$C = 90$

$C = \dfrac{\pi}{2}$

34. (D)



Let height of the lamp post (AB) = h m

In ΔABC

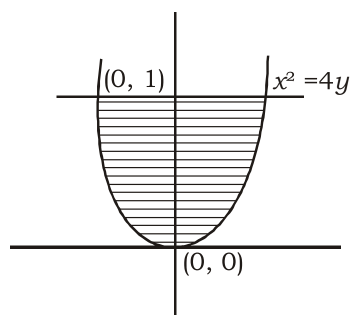
$$\tan 60^\circ = \frac{AB}{AC}$$

$$\sqrt{3} = \frac{h}{50}$$

$$h = 50\sqrt{3}$$

height of the lamp post $= 50\sqrt{3}\ m$

35. (C)



Parabola

$x^2 = 4y$

$4a = 4$

$a = 1$

$\Rightarrow \quad x = 2\sqrt{y}$

$$\text{Area} = 2\int_0^1 x\,dy$$

$$= 2\int_0^1 2\sqrt{y}\,dy$$

$$= 4 \times \frac{2}{3}\left[y^{\frac{3}{2}}\right]_0^1$$

$$= \frac{8}{3} \text{ square unit.}$$

36. (A) $y = \frac{\ln x}{x} + \frac{e^x}{x}$

On differentiating both side w.r.t 'x'

$$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - (\ln x).1}{x^2} + \frac{x \times e^x - e^x.1}{x^2}$$

$$\frac{dy}{dx} = \frac{1 - \ln x + xe^x - e^x}{x^2}$$

$$\left(\frac{dy}{dx}\right)_{at\,x=1} = \frac{1 - \ln 1 + 1.e^1 - e^1}{(1)^2}$$

$$= \frac{1 - 0 + e - e}{1}$$

$$= 1$$

37. (C) $\lim_{n\to\infty} \frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{1^3 + 2^2 + 3^3 + \ldots + n^3}$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{n}{6}(n+1)(2n+1)}{\left[\frac{1}{2}n(n+1)\right]^2}$$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{n^3}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{\frac{n^4}{4}\left(1+\frac{1}{n}\right)^2}$$

$$\Rightarrow \lim_{n\to\infty} \frac{4\left(2+\frac{1}{n}\right)}{6n\left(1+\frac{1}{n}\right)}$$

$$\Rightarrow \frac{8}{\infty}$$

$$\Rightarrow 0$$

38 (A) Differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{3}{2}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{4}}$$

$$\left(\frac{d^3y}{dx^3}\right)^{3} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{2}}$$

$$\left(\frac{d^3y}{dx^3}\right)^{6} = \frac{d^2y}{dx^2}$$

degree = 6

(39-41) given that $I = \int_0^{\pi} \frac{x}{1-\sin x}\, dx$

Property IV $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^{\pi} \frac{(\pi - x)}{1 - \sin(\pi - x)}\, dx$$

$$I = \int_0^{\pi} \frac{\pi}{1-\sin x}\, dx - \int_0^{\pi} \frac{x}{1-\sin x}\, dx$$

$$I = \pi\int_0^{\pi} \frac{1}{1-\sin x}\, dx - I$$

$$2I = \pi\int_0^{\pi} \frac{1}{1-\sin x}\, dx \ldots\text{(i)}$$

$$2I = \pi\int_0^{\pi} \frac{(1+\sin x)}{(1-\sin x)(1+\sin x)}\, dx$$

$$2I = \pi\int_0^{\pi} \frac{1+\sin x}{1-\sin^2 x}\, dx$$

$$2I = \pi\int_0^{\pi} (\sec^2 x + \sec x.\tan x)\, dx$$

$$2I = \pi[\tan x + \sec x]_0^{\pi}$$

$2I = \pi[\tan \pi + \sec \pi - \tan 0 - \sec 0]$

$2I = \pi[0-1-0-1]$

$2I = -2\pi$

$I = -\pi \quad \ldots\text{(ii)}$

39. (A) From equation (i)

$$2I = \pi\int_0^{\pi} \frac{1}{1-\sin x}\, dx$$

$$\int_0^{\pi} \frac{1}{1-\sin x}\, dx = \frac{2}{\pi} I$$

$$\int_0^{\pi} \frac{1}{1-\sin x}\, dx = \frac{2}{\pi}(-\pi) \quad \text{[from equation (ii)]}$$

$$\int_0^{\pi} \frac{1}{1-\sin x}\, dx = -2$$

40. (B) $I = \int_0^{\pi} \frac{xdx}{1-\sin x} = \int_0^{\pi} \frac{(\pi - x)}{1-\sin x}\, dx$

$$-\pi = \int_0^{\pi} \frac{(\pi - x)}{(1-\sin x)}\, dx$$

$$\int_0^{\pi} \frac{(\pi - x)}{(1-\sin x)}\, dx = -\pi$$

41. (A) from equation (ii)

$I = -\pi$

42.(A) $e^{\frac{dy}{dx}} = x$

taking log

$\frac{dy}{dx}\log e = \log x$

$\frac{dy}{dx} = \log x$

On integrating

$y = \int \log x dx$

$y = \log x \int 1.dx - \int\left\{\frac{d}{dx}(\log x).\int 1dx\right\} dx$

$y = x\log x - \int \frac{1}{x}\times x\, dx$

$y = x\log x - x + c$

$y = x(\log x - 1) + c$

43. (D) Vectors $\lambda\hat{i} + (2-\lambda)\hat{j} + (1-2\lambda)\hat{k}$ and $(1+\lambda)\hat{i} + \lambda\hat{j} + 4\hat{k}$ are perpendicular.

Then $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$\lambda(1+\lambda) + (2-\lambda)\lambda + (1-2\lambda)\times 4 = 0$

$\lambda + \lambda^2 + 2\lambda - \lambda^2 + 4 - 8\lambda = 0$

$4 - 5\lambda = 0$

$\lambda = \frac{4}{5}$

44. (C) $I = \int_0^1\left(x^{\frac{1}{3}} + x\right)dx$

$I = \left[\frac{3x^{\frac{4}{3}}}{4} + \frac{x^2}{2}\right]_0^1$

$I = \left[\frac{3}{4} + \frac{1}{2} - 0 - 0\right]$

$I = \frac{5}{4}$

45. (A) $I = \int_0^{\frac{\pi}{2}} \frac{\phi\left(\frac{\pi}{2}-x\right)}{\phi(x) + \phi\left(\frac{\pi}{2}-x\right)} dx$ ...(i)

Property IV $\int_0^a f(x) = \int_0^a f(a-x)dx$

$I = \int_0^{\frac{\pi}{2}} \frac{\phi(x)}{\phi\left(\frac{\pi}{2}-x\right) + \phi(x)} dx$ ...(ii)

from equation (i) and equation (ii)

$2I = \int_0^{\frac{\pi}{2}} \frac{\phi(x) + \phi\left(\frac{\pi}{2}-x\right)}{\phi(x) + \phi\left(\frac{\pi}{2}-x\right)} dx$

$2I = \int_0^{\frac{\pi}{2}} 1\,.dx$

$2I = [x]_0^{\frac{\pi}{2}}$

$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

(46-49) : $f(x) = 3x^3 + 4x^2 f'(1) - 2x f''(2) + f'''(3)$ ...(i)

On differentiating both side *w.r.t* '*x*'

$f'(x) = 9x^2 + 8x f'(1) - 2f''(2)$ ...(ii)

again, differentiating

$f''(x) = 18x + 8f'(1)$ ...(iii)

$f'''(x) = 18$ ...(iv)

from equation (ii)

$f'(1) = 9 + 8f'(1) - 2f''(2)$

$7f'(1) - 2f''(2) + 9 = 0$ ...(v)

from equation (iii)

$f''(2) = 18\times 2 + 8f'(1)$

$f''(2) - 8f'(1) = 36$ ...(vi)

46. (A) from equation (v) and equation (vi)

$\boldsymbol{f'(1) = -7}$ and $f''(2) = -20$

47. (C) from equation ...(i)

$f(x) = 3x^3 + 4x^2(-7) - 2x(-20) + 18$

$f(x) = 3x^3 - 28x^2 + 40x + 18$

$f(1) = 3 - 28 + 40 + 18$

$f(1) = 33$

48. (A) from equation (iv), $f'''(x) = 18$

$f'''(15) = 18$

49. (A) from equation (iv)

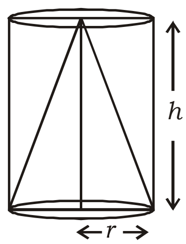
$f'''(x) = 18$

On differentiating w.r.t '*x*'

$f''''(x) = 0$

$f''''(2) = 0$

50. (C)



Given that
height of the cylinder (h) = $2r$
radius of cylinder = r
then
radius of cone = r
height of the cone = $2r$
then $l^2 = h^2 + r^2$
$l^2 = (2r)^2 + r^2$
$l^2 = 5r^2$
$l = \sqrt{5}\, r$
lateral surface area = $\pi r l$
$= \pi \times r \times \sqrt{5}\, r$
$= \sqrt{5}\, \pi r^2$

51. (B) Volume of cone = $\frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi \times r^2 \times 2r$
$= \frac{2}{3}\pi r^3$
Volume of cylinder = $\pi r^2 h$
$= \pi r^2 \times 2r$
$= 2\pi r^3$

$$\frac{\text{Volume of cone}}{\text{Volume of cylinder}} = \frac{\frac{2}{3}\pi r^3}{2\pi r^3}$$
$= \frac{1}{3}$
= 1 : 3

52. (D) (I) $\sin\frac{\pi}{6} = \frac{1}{2}$

(II) $\sin\frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right)$
$= \sin\frac{\pi}{4}$
$= \frac{1}{\sqrt{2}}$

(III) $\sin\frac{7\pi}{4} = \sin\left(2\pi - \frac{\pi}{4}\right)$
$= -\sin\frac{\pi}{4}$
$= -\frac{1}{\sqrt{2}}$

(IV) $\sin\frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right)$
$= \sin\frac{\pi}{3}$
$= \frac{\sqrt{3}}{2}$

then $-\frac{1}{\sqrt{2}} < \frac{1}{2} < \frac{1}{\sqrt{2}} < \frac{\sqrt{3}}{2}$
III < I < II < IV

53. (D) $x\sin\theta + y\cos\theta = 3$
slope $m = -\tan\theta$
and line $\sqrt{3}\,x + y = 5$
slope $m = -\sqrt{3}$
line $x\sin\theta + y\cos\theta = 3$ and line $\sqrt{3}\,x + y = 5$ are parallel to each other
then
$-\tan\theta = -\sqrt{3}$
$\tan\theta = \tan\frac{\pi}{3}$
$\theta = \frac{\pi}{3}$

54. (C) Planes $x + 2y - z = 5$
$a_1 = 1,\ b_1 = 2,\ c_1 = -1$
and $2x + y + z = 6$
$a_2 = 2,\ b_2 = 1,\ c_2 = 1$
angle between the planes

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_{-1}^2 (-1)^2}}$$

$$= \frac{1\times 2 + 2(1) + (-1)(1)}{\sqrt{(1)^2 + (2)^2 + (-1)^2}\sqrt{(2)^2 + (1)^2 + (1)^2}}$$

$$= \frac{3}{\sqrt{6}\sqrt{6}}$$

$\cos\theta = \frac{3}{6}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

55. (A) In general equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

major axis = $2a$

and latus rectum = $\frac{2b^2}{a}$

according to the equation

latus rectum = $\frac{1}{3}$ × major axis

$\frac{2b^2}{a} = \frac{1}{3} \times 2a$

$\frac{b^2}{a^2} = \frac{1}{3}$

then $e^2 = 1 - \frac{b^2}{a^2}$

$e^2 = 1 - \frac{1}{3}$

$e^2 = \frac{2}{3}$

$e = \sqrt{\frac{2}{3}}$

56. (C) Two dice are thrown.

total sample space $n(S) = 6 \times 6 = 36$

sum faces equals or exceeds 8.

event

(6, 2), (2, 6), (5, 3), (3, 5) (4, 4) for 8

(6, 3), (3, 6), (5, 4), (4, 5) for 9

(6, 4), (4, 6), (5, 5) for 10

(6, 5), (5, 6) for 11

(6, 6) for 12

$n(E) = 15$

$P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

57. (D) Given that

$\vec{a} = 2\hat{i} + 3\hat{j} + 3\hat{k},\ \vec{b} = \hat{i} - \hat{j} + 4\hat{k},$

$\vec{c} = \hat{i} + \hat{j} + \hat{k}$

$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$

$\Rightarrow \vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$

$\Rightarrow \vec{a} \times \vec{b} - \vec{c} \times \vec{a} + \vec{b} \times \vec{c} - \vec{a} \times \vec{b} + \vec{c} \times \vec{a} - \vec{b} \times \vec{c}$

$\Rightarrow 0$

58. (C) $1.8 + 0.18 + 0.018 + \ldots\infty$

$a = 1.8,\ r = \frac{1}{10}$

sum of G.P. $(S_n) = \frac{a}{1-r}$ $[r < 1]$

$= \frac{1.8}{1-\frac{1}{10}}$

$= \frac{1.8 \times 10}{9}$

$= 0.2 \times 10$

$= 2$

59. (A)

2	43	
2	21	1
2	10	1
2	5	0
2	2	1
2	1	0
	0	1

$(43)_{10} = (101011)_2$

60. (A) $C(29, 3r) = C(29, r-3)$

${}^{29}C_{3r} = {}^{29}C_{r-3}$

$3r + r - 3 = 29$ $\left[\because {}^nC_r = {}^nC_s \text{ then } r + s = n\right]$

$4r = 32$

$r = 8$

61.(C) $x = 3 + 3^{\frac{1}{3}} + 3^{\frac{2}{3}}$

$(x-3)^3 = \left(3^{\frac{1}{3}} + 3^{\frac{2}{3}}\right)^3$

$x^3 - 27 - 3x \times 3(x-3) = 3 + 9 + 3 \times 3\left(3^{\frac{1}{3}} + 3^{\frac{2}{3}}\right)$

$x^3 - 27 - 9x^2 + 27x = 12 + 9(x-3)$

$x^3 - 9x^2 + 27x - 27 = 12 + 9x - 27$

$x^3 - 9x^2 + 18x = 12$

$x^3 - 9x^2 + 18x + 10 = 22$

62. (A) $\begin{vmatrix} 1 & 1 & 1+a \\ 1+b & 1 & 1 \\ 1 & 1+c & 1 \end{vmatrix} = 0$

$abc \begin{vmatrix} \frac{1}{a} & \frac{1}{a} & \frac{1}{a}+1 \\ \frac{1}{b}+1 & \frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c}+1 & \frac{1}{c} \end{vmatrix} = 0$

$abc \neq 0$

$R_1 \to R_1 + R_2 + R_3$

$\begin{vmatrix} \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 \\ \frac{1}{b}+1 & \frac{1}{b} & \frac{1}{b} \\ \frac{1}{c}-1 & \frac{1}{c}+1 & \frac{1}{c} \end{vmatrix} = 0$

$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b}+1 & \frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c}+1 & \frac{1}{c} \end{vmatrix} = 0$

$C_2 \to C_2 - C_1, \; C_3 \to C_3 - C_1$

$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right) \begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b}+1 & -1 & -1 \\ \frac{1}{c} & 1 & 0 \end{vmatrix} = 0$

$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right) [1(0 + 1) + 0 + 0] = 0$

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} + 1 = 0$

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = -1$

63. (B) $-1 \quad 2 \quad 3$
$\uparrow \quad \uparrow \quad \uparrow$

$4 \leftarrow \begin{vmatrix} -a & 2x & 3p \\ -4b & 8y & 12q \\ -c & 2z & 3r \end{vmatrix} = \lambda \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$

$\Rightarrow -24 \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} = \lambda \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$

then $\lambda = -24$

64. (A) Equation

$xdx + ydy = 0$

On intergrating

$\int xdx + \int ydy = \int 0$

$\frac{x^2}{2} + \frac{y^2}{2} = c$

$x^2 + y^2 = 2c$

The equation represents a family of circles.

65. (A) equation

$x + 2(x-3)^{-1} = 3 + 2(x-3)^{-1}$

$x + \frac{2}{x-3} = 3 + \frac{2}{x-3}$

$\frac{x^2 - 3x + 2}{x-3} = \frac{3x - 9 + 2}{x-3}$

$x - 3 \neq 0, \; x^2 - 3x + 2 = 3x - 9 + 2$

$\boldsymbol{x \neq 3}, \; x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

$x = 3$

then equation has no roots.

66. (D) Determinant $\begin{vmatrix} 1 & 2 & 0 \\ 4 & 3 & 6 \\ -1 & 2 & 4 \end{vmatrix}$

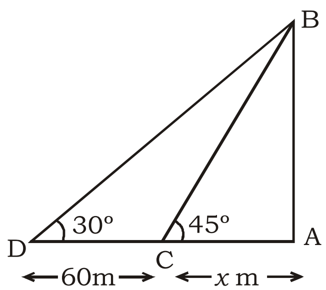
Co-factor of element (3) = C_{22}

$= (-1)^{2+2} \begin{vmatrix} 1 & 0 \\ -1 & 4 \end{vmatrix}$

$= 1(4 - 0)$

$= 4$

67. (C)



Let the breadth of the river (AC) = x cm

In ΔABC

$\tan 45 = \frac{AB}{AC}$

$1 = \frac{AB}{x}$

$AB = x$

In ΔABD

$\tan 30° = \frac{AB}{AD}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+60}$

$x + 60 = x\sqrt{3}$

$60 = x(\sqrt{3} - 1)$

$x = \frac{60}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$x = 30(\sqrt{3} + 1)$

breadth of the river = $30(\sqrt{3} + 1)$ m

68. (C) $\tan 75° = \tan(45° + 30°)$

$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° . \tan 30°}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \frac{4+2\sqrt{3}}{2}$

$= 2 + \sqrt{3}$

69. (B)

70. (A) $n(S) = 26$

$n(E) = {}^4C_1 = 4$

$P(E) = \frac{n(E)}{n(S)} = \frac{4}{26}$

$= \frac{2}{13}$

71. (C) $\frac{dy}{dx} = e^{y-x}\left(e^{x-y} + e^x\right)$

$\frac{dy}{dx} = 1 + e^y$

$\frac{dy}{1+e^y} = dx$

$\frac{e^{-y}dy}{e^{-y}\left(1+e^y\right)} = dx$

$\frac{-e^{-y}dx}{\left(e^{-y}+1\right)} = -dx$

On integrating

$\log(e^{-y} + 1) = -x + \log c$

$\log\left(\frac{1+e^y}{e^y}\right) = -x + \log c$

$\log(1 + e^y) - y = -x + \log c$

$y - x = \log\left(\frac{1+e^y}{c}\right)$

72. (C) $n(S) = 52$

a king or a queen

$n(E) = 4 + 4 = 8$

$P(E) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$

73. (D) $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{5}$, $P\left(\frac{A}{B}\right) = \frac{1}{4}$

then $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\frac{1}{4} = \frac{P(A \cap B)}{\frac{1}{5}}$

$P(A \cap B) = \frac{1}{4} \times \frac{1}{5}$

$P(A \cap B) = \frac{1}{20}$

$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$P\left(\frac{B}{A}\right) = \frac{\frac{1}{20}}{\frac{2}{3}}$

$P\left(\frac{B}{A}\right) = \frac{3}{40}$

74.(C)There are 40 women and 30 men employees.
Average salary of men = 3050
total salary of men = 3050 × 30
= 91500
Average salary of men and women = 4550
total salary of men and women = 70 × 4550
= 318500
total salary of women = 318500 – 91500
= 227000

average salary of women = $\frac{227000}{40}$
= 5675

75. (B) numbers 13, 15, 17, 91, 21

mean $(\bar{x}) = \frac{13+15+17+19+21}{5}$

$= \frac{85}{5} = 17$

Standard deviation (σ) = $\sqrt{\frac{\sum_{i=1}^{n}(x_1 - \bar{x})^2}{n}}$

$= \sqrt{\frac{(13-17)^2+(15-17)^2+(17-17)^2+(19-17)^2+(21-17)^2}{5}}$

$= \sqrt{\frac{16+4+0+4+16}{5}}$

$= \sqrt{\frac{40}{5}}$

$= \sqrt{8}$
= 2.8

76.(C) Given
vertex of parabola $(h, k) = (2, 3)$ and its focus $(h, a + k) = (2, 1)$
$a + k = 1$
$a + 3 = 1$
$a = -2$
x - co-ordinate of vertex and focus are same.
So axis of parabola is parallel to y-axis.
Thus equation of parabola
$(x - h)^2 = 4a(y - k)$
$(x - 2)^2 = -8(y - 3)$

77. (A)

P :	71	65	83	53	18	23
Q :	82	83	24	43	44	48

average score of P

$= \frac{71+65+83+53+18+23}{6}$

$= \frac{313}{6}$

= 52.167
Average score of Q

$= \frac{82+83+24+43+44+48}{6}$

$= \frac{324}{6}$

= 54
average of scores of P and Q are not same but Q is consistent.

78. (C) Hyperbola
$3x^2 - 2y^2 = 6$

$\frac{x^2}{2} - \frac{y^2}{3} = 1$

$a^2 = 2, \quad b^2 = 3$

$a = \sqrt{2}, \quad b = \sqrt{3}$

$e^2 = 1 + \frac{b^2}{a^2}$

$e^2 = 1 + \frac{3}{2}$

$e^2 = \frac{5}{2} \Rightarrow e = \frac{\sqrt{5}}{\sqrt{2}}$

foci = $(\pm ae, 0)$

$= \left(\pm\sqrt{2} \times \frac{\sqrt{5}}{\sqrt{2}}, 0\right)$

$= (\pm\sqrt{5}, 0)$

79. (C) $I = \int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$

Let $\sqrt{x} = t$

$$\frac{1}{2}\sqrt{x}\,dx = dt$$

$$\sqrt{x}\,dx = 2dt$$

$I = \int 2\cos t\,dt$

$I = 2\sin t + c$

$I = 2\sin\sqrt{x} + c$

80. (D) $x = 3t^2$, $y = \frac{t^2}{4}$

$\frac{dx}{dt} = 6t$, $\frac{dy}{dt} = \frac{2t}{4}$

$\frac{dt}{dx} = \frac{1}{6t}$, $\frac{dy}{dt} = \frac{t}{2}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{t}{2} \times \frac{1}{6t}$

$\frac{dy}{dx} = \frac{1}{12}$

$\frac{d^2y}{dx^2} = 0$

81. (B) $y = a^{x\log_a \cos x}$

$y = a^{\log_a (\cos x)^x}$

$y = (\cos x)^x$

taking log

$\log y = x\log\cos x$

On differentiating both w.r.t. 'x'.

$\frac{1}{y}\frac{dy}{dx} = x.\frac{1}{\cos x}(-\sin x) + \log(\cos x).1$

$\frac{dy}{dx} = y(-x\tan x + \log\cos x)$

$\frac{dy}{dx} = y(\log\cos x - x\tan x)$

82. (C) The combined equation of the pair of tangent drawn from (–1, 3) to the ellipse $2x^2 + 3y^2 = 6$ is

$SS' = T^2$

$(2x^2 + 3y^2 - 6)\,(2(-1)^2 + 3(3)^2 - 6)$
$= (2x(-1) + 3y\,(3) - 6)^2$

$\Rightarrow (2x^2 + 3y^2 - 6)\,(2 + 27 - 6) = (-2x + 9y - 6)^2$

On solving

$\Rightarrow 42x^2 - 12y^2 + 36xy - 24x + 108y - 174 = 0$

$a = 42,\ b = -12,\ h = 18$

The angle between the lines

$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

$= \frac{2\sqrt{(18)^2 - 42(-12)}}{42-12} = \frac{2\sqrt{828}}{30} = \frac{2\times 6\sqrt{23}}{30}$

$\theta = \tan^{-1}\left(\frac{2\sqrt{23}}{5}\right)$

83. (A) Differential equation

$\sin x\frac{dy}{dx} + y\cos x = 2$

$\frac{dy}{dx} + (\cot x)y = 2\,\text{cosec}\,x$

On comparing general equation

$\frac{dy}{dx} + Py = Q$

$P = \cot x,\ Q = 2\,\text{cosec}\,x$

I.F. $= e^{\int P dx}$

I.F. $= e^{\int \cot x dx}$

$= e^{\log\sin x}$

$= \sin x$

solution of differential equation

$y \times I.F. = \int Q \times I.F.\,dx$

$y \times \sin x = \int 2\ \text{cosec}\,x \times \sin x\,dx$

$y \times \sin x = \int 2\times 1\,dx$

$y \times \sin x = 2x + c$

$y = 2x.\text{cosec}x + \text{c.cosec}x$

84. (C) Let $y = \sqrt{6\sqrt{6\sqrt{6\ldots\infty}}}$

$y = \sqrt{6y}$

$y^2 = 6y$

$y^2 - 6y = 0$

$y(y - 6) = 0$

$y \neq 0,\ y = 6$

85. (A) $z = \frac{1+i}{2-i} + \frac{1-i}{2i+i}$

$z = \frac{1+i}{(2-i)} \times \frac{(2+i)}{(2+i)} + \frac{1-i}{2i+1} \times \frac{2i-1}{2i-1}$

$z = \frac{2+2i+i+i^2}{4-i^2} + \frac{2i-2i^2-1+i}{4i^2-1}$

$z = \frac{1+3i}{5} + \frac{3i+1}{-5}$

$z = 0$

then

$z\bar{z} - z^2 = 0$

86. (D) We know that

$-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

then

$-\sqrt{8^2+15^2} \le 8\sin\theta + 15\cos\theta \le \sqrt{8^2+15^2}$

$-\sqrt{289} \le 8\sin\theta + 15\operatorname{cosec}\theta \le \sqrt{289}$

$-17 \le 8\sin\theta + 15\cos\theta \le 17$

$-17 \le P \le 17$

87. (C)

class	x	f	$f \times x$
5 – 15	10	8	80
15 – 25	20	9	180
25 – 35	30	11	330
35 – 45	40	13	520
45 – 55	50	9	450

$\Sigma f = 50, \ \Sigma f \times x = 1560$

Mean $= \frac{\Sigma f \times x}{\Sigma f}$

$= \frac{1560}{50}$

$= 31.2$

88. (B) $I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} x^2 \sin^3 x \, dx$

$I = 0$

$\left[\because \int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is even} \\ 0 & ,\text{if } f(x) \text{ is odd} \end{cases}\right]$

89. (C) $I = \int_0^{20\pi} |\sin x| \, dx$

$|\sin x|$ is periodic with period π

$\therefore \ I = 20\int_0^{\pi} \sin x \, dx$

$= 20[-\cos x]_0^{\pi} dx$

$I = 20(1 + 1) = 40$

90. (A) $f(x) = 3x^2 - 12x + 5$ ...(i)

$f'(x) = 6x - 12$

$f''(x) = 6$ ...(ii)

for maxima or manima

$f'(x) = 0$

$6x - 12 = 0$

$x = 2$

On putting $x = 2$ in equation (ii)

$f''(2) = 6$ (minima)

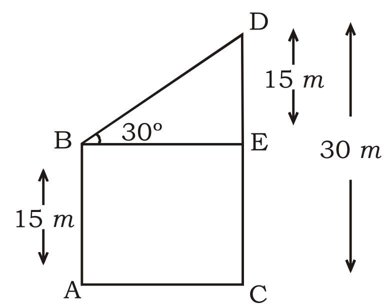
On putting $x = 2$ in equation (i)

minimum value $= 3(2)^2 - 12 \times 2 + 5$

$= 12 - 24 + 5$

$= -7$

91. (C)



In ΔBED

$\tan 30^\circ = \frac{DE}{BE}$

$\frac{1}{\sqrt{3}} = \frac{15}{BE}$

$BE = 15\sqrt{3}$

distance between towers $= 15\sqrt{3}$ m

92. (A) a cricket team of 11 players be chosen out of a batch of 15 players so that one player of the team is never included.

number of ways $= {}^{15-1}C_{11}$

$= {}^{14}C_{11}$

$= \frac{14!}{11!3!}$

$= 364$

93. (C) $\cos^2 5^\circ + \cos^2 10^\circ + \cos^2 15^\circ + \ldots + \cos^2 90^\circ$

$\Rightarrow \cos^2 5^\circ + \cos^2 10^\circ + \ldots + \cos^2 80^\circ + \cos^2 85^\circ + 0$

$\Rightarrow \cos^2 5^\circ + \cos^2 10^\circ + \ldots + \sin^2 10^\circ + \sin^2 5^\circ$

$\Rightarrow (\cos^2 5^\circ + \sin^2 5^\circ) + (\cos^2 10^\circ + \sin^2 10^\circ) + \ldots + (\cos^2 40^\circ + \sin^2 40^\circ) + \cos^2 45^\circ$

$= (1 + 1 + \ldots 8 \text{ times}) + \frac{1}{2}$

$= 8 + \frac{1}{2}$

$= \frac{17}{2}$

94. (C) In the expansion of $\left[3x^2 - \frac{1}{3\sqrt{x}}\right]^{10}$

general term

$$T_{r+1} = {}^{10}C_r\,(3x^2)^{10-r}\left(\frac{-1}{3\sqrt{x}}\right)^r$$

$$= {}^{10}C_r\,3^{10-r}\left(\frac{-1}{3}\right)^r x^{20-2r-\frac{r}{2}}$$

$$20 - 2r - \frac{r}{2} = 0$$

$$20 = \frac{5r}{2}$$

$$r = 8$$

$$T_{8+1} = T_9 = {}^{10}C_8\,3^2\left(\frac{-1}{3}\right)^8$$

$$= \frac{10!}{8!2!} \times 9 \times 3^{\frac{1}{8}}$$

$$= \frac{5}{81}$$

constant term of the expansion = $\frac{5}{81}$

95. (C) Word 'MEERUT'

word formed starting with the letter 'E' = 5!

= 5 × 4 × 3 × 2 × 1

= 120

word formed starting with the letter 'M' =

$\frac{5!}{2!} = 60$

Ist word formed starting with the letter '*R*'

⇒ REEMTU

IInd word formed starting with the letter '*R*'

⇒ REEMUT

96. (C) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 0 \end{bmatrix}$

$2X - 5A = 0$

$2X = 5A$

$$2X = 5\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 0 \end{bmatrix}$$

$$2X = \begin{bmatrix} 5 & 10 \\ 15 & 20 \\ -5 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{5}{2} & 5 \\ \frac{15}{2} & 10 \\ \frac{-5}{2} & 0 \end{bmatrix}$$

97. (C) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 2 & 4 \end{bmatrix}$

$$BA = \begin{bmatrix} -1 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & -2 \\ 2 & 8 \end{bmatrix}$$

$$|BA| = \begin{vmatrix} -1 & -2 \\ 2 & 8 \end{vmatrix} = -8 + 4 = -4$$

Co-factor of (BA)

$C_{11} = (-1)^{1+1}(8)$, $C_{12} = (1)^{1+2}(2)$

$= 8$ $\quad = -2$

$C_{21} = (-1)^{2+1}(-2)$, $C_{22} = (-1)^{2+2}(-1)$

$= 2$ $\quad = -1$

$$C = \begin{bmatrix} 8 & -2 \\ 2 & -1 \end{bmatrix}$$

$$\text{Adj}(BA) = C^T = \begin{bmatrix} 8 & 2 \\ -2 & -1 \end{bmatrix}$$

$$(BA)^{-1} = \frac{\text{Adj}(BA)}{|BA|} = \frac{\begin{bmatrix} 8 & 2 \\ -2 & -1 \end{bmatrix}}{-4} = \begin{bmatrix} -2 & \frac{-1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

$$(BA)^{-1} = A^{-1}B^{-1} = \begin{bmatrix} -2 & \frac{-1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

98. (A) $\frac{1+\cos\theta}{1-\cos\theta} = 3$

$$\frac{2\cos^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = 3$$

$\cot^2 \frac{\theta}{2} = 3$

$\cot^2 \frac{\theta}{2} = \cot^2 \frac{\pi}{6}$

$\tan^2 \frac{\theta}{2} = \tan^2 \frac{\pi}{6}$

$\frac{\theta}{2} = n\pi \pm \frac{\pi}{6}$

$\theta = 2n\pi \pm \frac{\pi}{3}$

99. (C) $\lim_{x\to\infty} \left(\frac{x+b}{x+a}\right)^{x+a}$

$= \lim_{x\to\infty} \left(\frac{x+a+b-a}{x+a}\right)^{x+a}$

$= \lim_{x\to\infty} \left[\left(1+\frac{b-a}{x+a}\right)^{\frac{x+a}{b-a}}\right]^{b-a}$

$= e^{b-a}$

100. (C) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$

then

$\lim_{x\to 0^+} f(x) = f(0)$

$\lim_{h\to 0} f(0+h) = k$

$\lim_{h\to 0} \frac{\sin(0+h)}{0+h} = k$

$\lim_{h\to 0} \frac{\sin h}{h} = k$

$1 = k \qquad \left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$

101. (C) $I = \int \frac{(x+3)}{(x^2+5x+6)\sqrt{x+2}}\, dx$

$I = \int \frac{(x+3)}{(x+3)(x+2)\sqrt{x+2}}\, dx$

$I = \int \frac{1}{(x+2)^{\frac{3}{2}}}\, dx$

$I = \frac{(x+2)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + c$

$I = -2(x+2)^{\frac{-1}{2}} + c$

$I = \frac{-2}{\sqrt{x+2}} + c$

102. (B) $I = \int e^x \left[\frac{x}{(x+1)^2}\right] dx$

$I = \int e^x \left[\frac{x+1-1}{(x+1)^2}\right] dx$

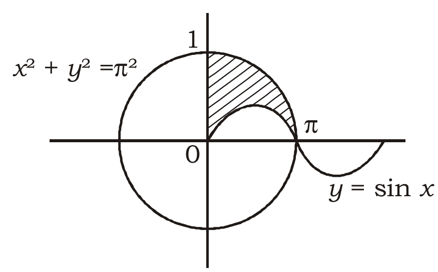
$I = \int e^x \left[\frac{1}{(x+1)} - \frac{1}{(x+1)^2}\right] dx$

$I = e^x \frac{1}{x+1} + c$

$\left[\because \int e^x \left[f(x) + f'(x)\right] dx = e^x f(x) + c\right]$

$I = \frac{e^x}{x+1} + c$

103. (C)



curve $x^2 + y^2 = \pi^2$

$y_1 \Rightarrow y = \sqrt{\pi^2 - x^2}$

and $y_2 \Rightarrow y = \sin x$

Area $= \int_0^{\pi} (y_1 - y_2)\, dx$

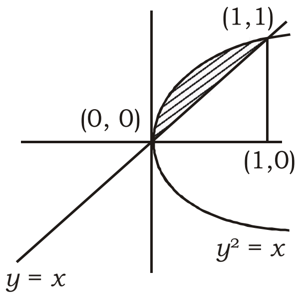
$= \int_0^{\pi} \left[\sqrt{\pi^2 - x^2} - \sin x\right] dx$

$= \left[\frac{1}{2} x\sqrt{\pi^2 - x^2} + \frac{1}{2}\pi^2 \sin^{-1}\frac{x}{\pi} + \cos x\right]_0^{\pi}$

$= \left[\left(0 + \frac{1}{2}\pi^2 \sin^{-1}(1) - 1\right) - (0 + 0 + \cos 0)\right]$

$= \frac{1}{2}\pi^2 \times \frac{\pi}{2} - 1 - 1 \Rightarrow \frac{\pi^3}{4} - 2$

104. (B)



Parabola $y^2 = x$

$y_1 \Rightarrow y = \sqrt{x}$ ...(i)

line $y = x$

$y_2 \Rightarrow y = x$ ...(ii)

from equation (i) and equation (ii)

$x = 0, \quad x = 1$

$y = 0, \quad y = 1$

Area = $\int_0^1 (y_1 - y_2)\, dx$

$= \int_0^1 (\sqrt{x} - x)\, dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^1$

$= \left(\frac{2}{3} - \frac{1}{2}\right) - (0 - 0)$

Area = $\frac{1}{6}$ square unit.

105. (A) $\frac{dy}{dx} = \frac{y}{x - 3y^3}$

$\frac{dx}{dy} = \frac{x - 3y^3}{y}$

$\frac{dx}{dy} - \frac{1}{y}\, x = -3y^2$

On comparing general equation

$\frac{dx}{dy} + Px = Q$

[where P, Q are the function of y]

$P = -\frac{1}{y}$, $Q = -3y^2$

I.F. $= e^{\int P dy}$

$= e^{\int -\frac{1}{y} dy}$

$= e^{-\log y}$

$= \frac{1}{y}$

Solution of differential equation

$x \times I.\ F. = \int Q \times \text{I.F.}\ dy$

$x \times \frac{1}{y} = \int -3y^2 \times \frac{1}{y}\, dy$

$\frac{x}{y} = -3\int y\, dy$

$\frac{x}{y} = -3\frac{y^2}{2} + \frac{c}{2}$

$2x = -3y^3 + cy$

106. (A) Point $(at_1^2, 2at_1)$, $(a, 0)$ and $(at_2^2, 2at_2)$ are collinear.

then

$$\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ a & 0 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix} = 0$$

$at_1^2 (0 - 2at_2) - 2at_1(a - at_2^2) + 1(2a^2t_2 - 0) = 0$

$-2a^2t_1^2 t_2 - 2a^2t_1 + 2a^2t_1t_2^2 + 2a^2t_2 = 0$

$2a^2t_1t_2^2 - 2a^2t_1^2 t_2 - 2a^2t_1 + 2a^2t_2 = 0$

$2a^2t_1t_2 (t_2 - t_1) + 2a^2 (t_2 - t_1) = 0$

$(t_2 - t_1)\ 2a^2 (t_1 t_2 + 1) = 0$

$t_1t_2 = -1$

107. (C) Circle

$x^2 + y^2 - 4x + 6y + 5 = 0$

$g = -2, f = 3, c = 5$

centre $(-g, -f) = (2, -3)$

equation of circle whose centre $(2, -3)$

$x^2 + y^2 - 4x + 6y + k = 0$ ...(i)

circle (i) passing throught the point $(1, -3)$

$(1)^2 + (-3)^2 - 4 \times 1 + 6(-3) + k = 0$

$1 + 9 - 4 - 18 + k = 0$

$k = 12$

from equation (i)

equation of circle

$x^2 + y^2 - 4x + 6y + 12 = 0$

108. (B) equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent an ellipse

if $\Delta \neq 0$, $ab - h^2 > 0$, $a + b = 0$

109. (C)

110. (A) numbers $0, 1, 2, 3, \ldots n$

$$\text{Average} = \frac{0^3+1^3+2^3+\dots n^3}{n+1}$$

$$= \frac{\frac{1}{2}n(n+1)\times\frac{1}{2}n(n+1)}{n+1}$$

$$= \frac{1}{4}n^2(n+1)$$

111. (C) Given that

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + \hat{j} + \hat{k}$

$\vec{a} - t\vec{b} = (2\hat{i} + 3\hat{j} - \hat{k}) - t(3\hat{i} - 4\hat{j} + 2\hat{k})$

$\vec{a} - t\vec{b} = (2-3t)\hat{i} + (3+4t)\hat{j} + (-1-2t)\hat{k}$

$(\vec{a} - t\vec{b})$ and $\vec{c}$ are perpendicular to each other.

then $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$(2-3t)1 + (3+4t)1 + (-1-2t)1 = 0$

$-t + 4 = 0$

$t = 4$

112. (A) Let equation of sphere

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ ...(i)

it passes through (0, 0, 0)

$d = 0$

it meets x-axis at (4, 0, 0)

$16 + 0 + 0 + 8u + 0 + 0 + 0 + 0 = 0$

$u = -2$

It meet y-axis at (0, –1, 0)

$0 + 1 + 0 + 0 + 2v(-1) + 0 + 0 = 0$

$$v = \frac{1}{2}$$

It meet z-axis at (0, 0, 3)

$0 + 0 + 9 + 0 + 0 + 2w(3) + 0 = 0$

$$w = -\frac{3}{2}$$

from equation (i)

equation of sphere

$x^2 + y^2 + z^2 - 4x + y - 3z = 0$

113. (B) Let $\vec{a} = \sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes an angle θ with y-axis. Then at makes an angle (90 –θ) with xz-plane.

Let $\vec{b} = 0\hat{i} + 1.\hat{j} + 0.\hat{k}$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{0+1+0}{\sqrt{2+1+1}\sqrt{1}}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

So the angle between the xz-plane and the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k} = \left(\frac{\pi}{2} - \theta\right)$

$$= \frac{\pi}{2} - \frac{\pi}{3}$$

$$= \frac{\pi}{6}$$

(114-116)

114. (B) $L_2 \Rightarrow \frac{x+3}{8} = \frac{y-4}{-4} = \frac{z-4}{-2}$

$$L_1 \Rightarrow \frac{x-4}{4} = \frac{y+5}{8} = \frac{y-6}{-2}$$

Direction Ratio of line L_2 is (8, –4, –2)

Direction Cosine $L_2 \Rightarrow < \frac{8}{\sqrt{8^2+(-4)^2+(-2)^2}}, \frac{-4}{\sqrt{8^2+(-4)^2+(-2)^2}}, \frac{-2}{\sqrt{8^2+(-4)^2+(-2)^2}} >$

$$\Rightarrow < \frac{4}{\sqrt{21}}, \frac{-2}{\sqrt{21}}, \frac{-1}{\sqrt{21}} >$$

115. (C) DR of L_1 = (4, 8, –2)

DR of L_2 = (8, – 4, –2)

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{4\times8+8(-4)+(-2)(-2)}{\sqrt{84}\sqrt{84}}$$

$$\cos\theta = \frac{4}{84}$$

$$\cos\theta = \frac{1}{21}$$

$$\theta = \cos^{-1}\left(\frac{1}{21}\right)$$

116. (A) Direction Ratio of L_1 = < 4, 8, –2 >

117. (C) curve

$4x^2 + 9y^2 - 16x + 18y - 11 = 0$

$4(x-2)^2 - 16 + 9(y+1)^2 - 9 - 11 = 0$

$4(x-2)^2 + 9(y+1)^2 = 36$

$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1$$

$$\frac{X^2}{9} + \frac{Y^2}{4} = 1 \text{ where } X = x-2$$

$Y = y + 1$

$a = 3,\ b = 2$

It is ellipse.

then

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$e^2 = 1 - \frac{4}{9}$$

$$e = \frac{\sqrt{5}}{3}$$

foci of ellipse $(X, Y) = (\pm ae, 0)$

$X = \pm ae$, $Y = 0$

$x - 2 = \pm 3 \times \frac{\sqrt{5}}{3}$, $y + 1 = 0$

$x = \pm\sqrt{5} + 2,$ $y = -1$

foci of the ellipse = $(2 \pm \sqrt{5}, -1)$

118. (A)

119. (C)

120. (A) $\frac{dx}{dy} - \sqrt{\frac{1-x^2}{1-y^2}} = 0$

$$\frac{dx}{dy} = \frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$$

$$\frac{dx}{\sqrt{1-x^2}} = \frac{dy}{\sqrt{1-y^2}}$$

On integrating

$$\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-y^2}}$$

$\sin^{-1} x = \sin^{-1} y + c$

$\sin^{-1} x - \sin^{-1} y = c$

NDA (MATHS) MOCK TEST - 80 (Answer Key)

1.	(C)	21.	(B)	41.	(A)	61.	(C)	81.	(B)	101.	(C)
2.	(C)	22.	(D)	42.	(A)	62.	(A)	82.	(C)	102.	(B)
3.	(B)	23.	(C)	43.	(D)	63.	(B)	83.	(A)	103.	(C)
4.	(B)	24.	(C)	44.	(C)	64.	(A)	84.	(C)	104.	(B)
5.	(C)	25.	(A)	45.	(A)	65.	(A)	85.	(A)	105.	(A)
6.	(B)	26.	(B)	46.	(A)	66.	(D)	86.	(D)	106.	(A)
7.	(B)	27.	(C)	47.	(C)	67.	(C)	87.	(C)	107.	(C)
8.	(C)	28.	(A)	48.	(A)	68.	(C)	88.	(B)	108.	(B)
9.	(A)	29.	(C)	49.	(A)	69.	(B)	89.	(C)	109.	(C)
10.	(B)	30.	(C)	50.	(C)	70.	(A)	90.	(A)	110.	(A)
11.	(C)	31.	(C)	51.	(B)	71.	(C)	91.	(C)	111.	(C)
12.	(C)	32.	(A)	52.	(D)	72.	(C)	92.	(A)	112.	(A)
13.	(D)	33.	(B)	53.	(D)	73.	(D)	93.	(C)	113.	(B)
14.	(C)	34.	(D)	54.	(C)	74.	(C)	94.	(C)	114.	(B)
15.	(A)	35.	(C)	55.	(A)	75.	(B)	95.	(C)	115.	(C)
16.	(B)	36.	(A)	56.	(C)	76.	(C)	96.	(C)	116.	(A)
17.	(C)	37.	(C)	57.	(D)	77.	(A)	97.	(C)	117.	(C)
18.	(A)	38	(A)	58.	(C)	78.	(C)	98.	(A)	118.	(A)
19.	(C)	39.	(A)	59.	(A)	79.	(C)	99.	(C)	119.	(C)
20.	(B)	40.	(B)	60.	(A)	80.	(D)	100.	(C)	120.	(A)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*